Mathematics 13 : Elementary Statistics

Unit 3 : Section 4

Lesson 35:
Hypothesis Testing Procedure for
Two Population Means

Example 3

Example 3

The reaction time of a test subject is measured by having them stand and place the palms of their hands on the surface of a table equally spaced on both sides of a reaction timing device. When initiated, the device will display a red light, and after some random interval, the light will change to green. The test subject is instructed to tap the screen of the device with their finger as soon as they see the green light appear.

Lesson 35 : Hypothesis Testing Procedure for Two Population Means

Example 3

The reaction time is the total number of milliseconds that transpires from the moment the light turns green (the stimulus) to the moment the test subject taps the screen of the device with their finger (the response).

Lesson 35 :

Hypothesis Testing Procedure for Two Population Means

Example 3

The reaction times for a random sample of 33 men produced a mean reaction time of 439.8 ms with a standard deviation of 31.7 ms.

The reaction times for a random sample of 37 women produced a mean reaction time of 480.1 ms with a standard deviation of 42.6 ms.

Lesson 35:

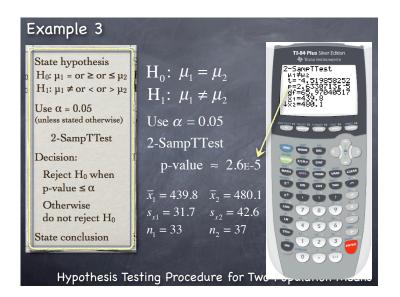
Hypothesis Testing Procedure for Two Population Means

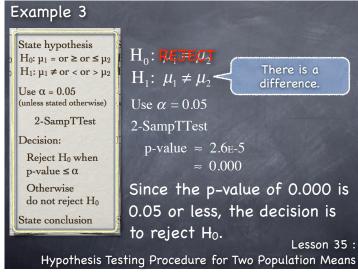
Example 3

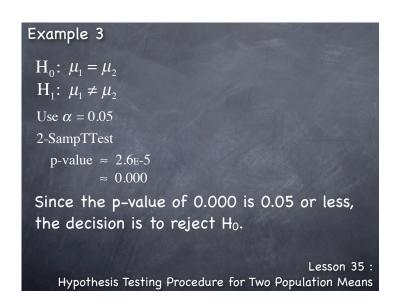
According to these results, is there evidence to clearly suggest that there is a difference in mean reaction time of men and women?

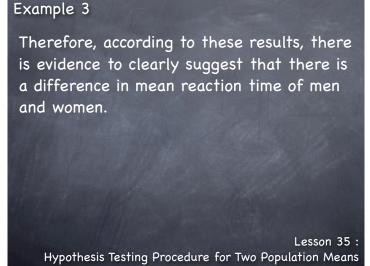
Lesson 35:

Hypothesis Testing Procedure for Two Population Means









Lesson 37:

Analyzing Data for Two Dependent Samples

Analyzing Data for Two Dependent Samples

Two dependent samples involves the collection of paired data values recorded from the same sample subjects.

Two dependent samples are generated when one sample of subjects is selected from the members of one population and two data values are collected from each of the subjects in the sample.

Lesson 37:

Analyzing Data for Two Dependent Samples

In contrast, two independent samples involves the collection of distinct data values recorded from separate sample subjects.

Two independent samples are generated when one sample of subjects is selected from the members of one population and another sample of subjects is selected from the members of a second population.

Lesson 37:

Analyzing Data for Two Dependent Samples
For instance,

in order to compare the mean GPA of students in High School and College using inferential statistics, a random sample of High School students and a random sample of College students was selected. The GPA for each of the randomly selected students was recorded.

Lesson 37:

Analyzing Data for Two Dependent Samples

The two samples of GPAs generated in this study are deemed independent since there were two populations (High School students and College students) from which the two separate samples were selected.

High School Students

3.44 3.30 3.83 4.00

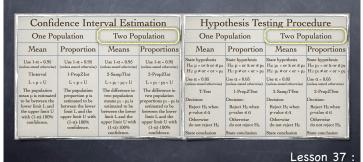
3.48 3.17 2.95 3.46

3.79 3.24 4.00 3.42

Lesson 37

Analyzing Data for Two Dependent Samples

In inferential statistics, two independent samples are analyzed using the two population methods.



Analyzing Data for Two Dependent Samples
For instance,

in order to compare the mean GPA of students in High School and College using inferential statistics, a random sample of College students was selected. For each of the randomly selected students, both their college GPA and High School GPA were recorded.

Lesson 37:

Analyzing Data for Two Dependent Samples

The two samples of GPAs generated in this study are deemed dependent since there was only one population (College students) from which two data values were recorded from each subject selected in the sample.

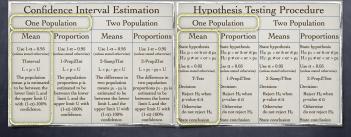
College Students									
College GPA	3.72	3.04	2.78	4.00	3.41	2.54			
High School GPA	3.92	3.17	2.87	4.00	3.25	2.86			
Student	1	2	3	4	5	6			

Lesson 37:

Lesson 37

Analyzing Data for Two Dependent Samples

In inferential statistics, two dependent samples are analyzed using the one population methods.



Lesson 37:

Analyzing Data for Two Dependent Samples

In order to analyze the data for two dependent samples using one population inferential statistics methods, the difference in the paired data values are calculated and used in the analysis.

College Students									
College GPA	3.72	3.04	2.77	4.00	3.41	2.54			
High School GPA				4.00					
Difference	20	13	09	0.00	0.16	32			
Difference	20	13	09	0.00	0.16	32			

Analyzing Data for Two Dependent Samples

When the first data value is greater than the second data value, the difference in the paired data values is greater than zero.

difference = data₁ - data₂ > 0

When the first data value equals the second data value, the difference in the paired data values equals zero.

difference = data₁ - data₂ $\stackrel{\mathbf{v}}{=} 0$

When the first data value is less than the second data value, the difference in the paired data values is less than zero.

difference = data₁ - data₂ $\stackrel{\mathbf{v}}{<}$ 0

Lesson 37:

Analyzing Data for Two Dependent Samples

It is important to distinguish between data that has been collected as two independent samples from data that has been collected as two dependent samples.

Two independent samples are analyzed using two population inferential statistics methods. Whereas, the difference in the two dependent samples is analyzed using one population inferential statistics methods.

Lesson 37:

Example 1

On average, does a person's dominant hand react quicker than their non-dominant hand?

In order to address this question statistically, a random sample of seven test subjects had the reaction time of both their dominant hand and non-dominant hand measured.

The following results were observed.

Lesson 37 : Analyzing Data for Two Dependent Samples

Example 1 Dominant Difference Hand Subject dependent samples 452 476 values were recorded 410 415 2 -5 from the same sample 3 388 381 7 463 516 -53 samples is calculated 479 461 18 and analyzed using one population 6 392 404 -12 inferential statistics 7 507 471 36

Lesson 37: Analyzing Data for Two Dependent Samples

Example 1

Assuming that people's reaction time reasonably follows a normal distribution, can one conclude from these sample results that, on average, a person's dominant hand reacts quicker than their non-dominant hand?

Lesson 37 : Analyzing Data for Two Dependent Samples

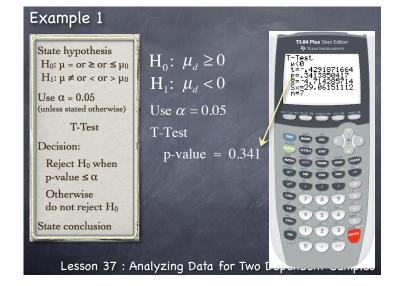
Example 1

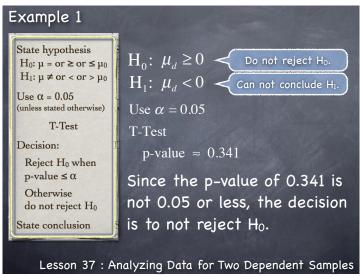
Assuming that people's reaction time reasonably follows a normal distribution, can one conclude from these sample results

that, on average, a person's dominant hand reacts quicker than their non-dominant hand?

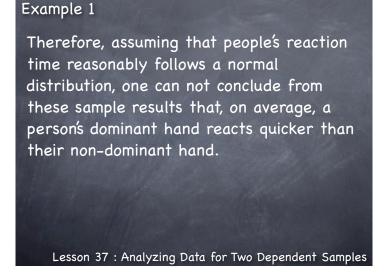
In order to apply inferential statistics methods which rely on the results of the Central Limit Theorem when the sample size is not sufficiently large, the population from which the random sample was selected must reasonably follow a normal distribution.

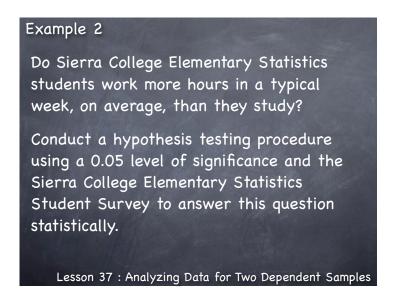
Lesson 37: Analyzing Data for Two Dependent Samples

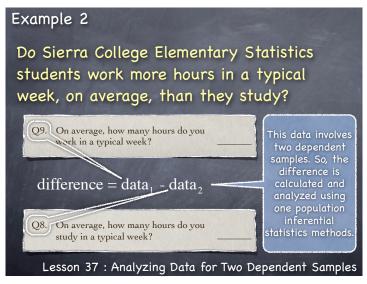


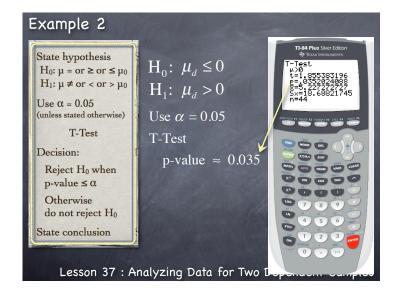


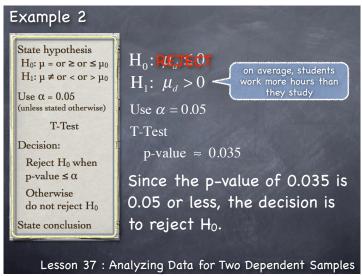
```
Example 1 H_0\colon \mu_d \geq 0 H_1\colon \mu_d < 0 Use \alpha = 0.05 T-Test p\text{-value} \approx 0.341 Since the p-value of 0.341 is not 0.05 or less, the decision is to not reject H_0.
```



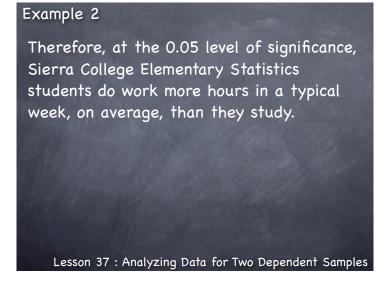




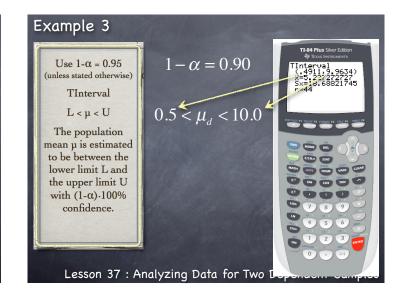


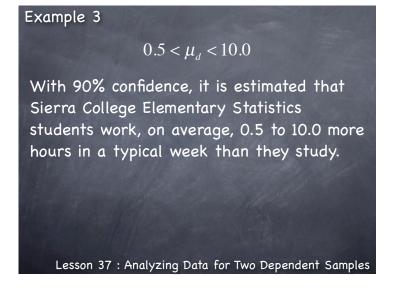


Example 2 $H_0\colon \mu_d \leq 0$ $H_1\colon \mu_d > 0$ Use $\alpha = 0.05$ T-Test $p\text{-value} \approx 0.035$ Since the p-value of 0.035 is 0.05 or less, the decision is to reject H_0 .



Estimate how many more hours, on average, Sierra College Elementary Statistics students work in a typical week than they study using a 90% confidence interval and the Sierra College Elementary Statistics Student Survey. Lesson 37: Analyzing Data for Two Dependent Samples







Your solutions should be clear, complete, and sufficiently detailed in order to demonstrate your understanding and communicate your reasoning and method of solving the problem.

Exercise 1

When 10 men and 10 women were randomly selected, the length of their right foot was measured in centimeters from the back of their heel to the tip of their toe. The results are summarized below.

Sample		Length (in cm) of Right Foot								
Men	25.6	27.9	28.2	27.5	27.0	26.8	29.1	28.6	25.8	27.4
Women	24.5	22.3	25.2	24.3	25.8	26.1	25.4	25.7	24.3	24.6

Do these results provide enough evidence to establish that the mean right foot length of men is longer than the mean right foot length of women? Conduct the hypothesis testing procedure under the assumption that the right foot length of both men and women reasonably follow a normal distribution.

Unit 3 : Section 4 Exercises

Exercise 2

When 10 men and 10 women were randomly selected, the length of their right foot was measured in centimeters from the back of their heel to the tip of their toe. The results are summarized below.

Sample		Length (in cm) of Right Foot								
Men	25.6	27.9	28.2	27.5	27.0	26.8	29.1	28.6	25.8	27.4
Women	24.5	22.3	25.2	24.3	25.8	26.1	25.4	25.7	24.3	24.6

Estimate the actual difference in the mean right foot length of men and women? Construct the confidence interval estimate under the assumption that the right foot length of both men and women reasonably follow a normal distribution. Express the estimate both symbolically and verbally.

A random sample of 10 people had the length of both their left foot and right foot measured in centimeters from the back of their heel to the tip of their toe. The results are displayed below.

		Length of Foot (in cm)								
Left Foot	24.1	23.5	28.6	27.8	26.9	25.8	26.6	25.1	29.3	25.3
Right Foot	23.9	23.2	28.8	27.8	27.0	25.5	26.7	24.9	29.7	25.0

Is there a significant difference in the length of a person's left foot and right foot on the average? Use α = 0.01 and assume that the length of a person's left foot and right foot both reasonably follow a normal distribution.

A sample of 8 people participated in a low calorie weight loss program for 16 weeks. Each of the participants had their weight (in pounds) recorded before they started the program and after they finished the program. The following results were obtained.

Participant	1	2	3	4	5	6	7	8
Weight Before	187	243	168	203	159	215	230	196
Weight After	172	222	151	175	146	207	206	177

At the 0.05 level of significance, test the claim that, on average, this 16 week low calorie weight loss program is effective? Assume that the amount of weight people lose after 16 weeks participating in this low calorie weight loss program reasonably follows a normal distribution.

A sample of 8 people participated in a low calorie weight loss program for 16 weeks. Each of the participants had their weight (in pounds) recorded before they started the program and after they finished the program. The following results were obtained.

Participant	1	2	3	4	5	6	7	8
Weight Before	187	243	168	203	159	215	230	196
Weight After	172	222	151	175	146	207	206	177

Estimate the mean amount of weight a person participating in this low calorie weight loss program can expect to lose at the end of 16 weeks. Assume that the amount of weight people lose after 16 weeks participating in this low calorie weight loss program reasonably follows a normal distribution. Express the estimate both symbolically and verbally.

The instructor of a CPR (cardiopulmonary resuscitation) training course administered a test designed to measure a person's understanding of the CPR procedure to the students both at the beginning and at the conclusion of the course. The student's post-training and pre-training CPR test scores are provided in the table below.

Student	1	2	3	4	5	6	7	8
Post-Training	19	21	16	23	24	21	25	22
Pre-Training	6	9	2	11	6	11	18	7

Use a 99% confidence interval to estimate the mean improvement in the CPR test score of students upon completion of this training course. Assume that the differences in the post-training and pre-training CPR test scores are normally distributed. Express the estimate both symbolically and verbally.

Your solutions should be clear, complete, and sufficiently detailed in order to demonstrate your understanding and communicate your reasoning and method of solving the problem.

Exercise 1

When 10 men and 10 women were randomly selected, the length of their right foot was measured in centimeters from the back of their heel to the tip of their toe. The results are summarized below.

Sample		Length (in cm) of Right Foot								
Men	25.6	27.9	28.2	27.5	27.0	26.8	29.1	28.6	25.8	27.4
Women	24.5	22.3	25.2	24.3	25.8	26.1	25.4	25.7	24.3	24.6

Do these results provide enough evidence to establish that the mean right foot length of men is longer than the mean right foot length of women? Conduct the hypothesis testing procedure under the assumption that the right foot length of both men and women reasonably follow a normal distribution.

 $H_0: \mu_1 \leq \mu_2$

 $H_1: \mu_1 > \mu_2$

Use $\alpha = 0.05$

2-SampTTest Inpt: Dsiz Stats List1:L1
List2:Lz Freq1:1
Freq2:1 μ1:≠μ2 <μ2 Συ Σ ↓Pooled: ≧© Yes

p-value
$$\approx 3.4E - 5 \approx 0.000$$

L1	L2	L3 3
25.6 27.9 28.2 27.5 27 26.8 29.1	50000845 40045665 20022000	
L3(1)=		

Since the p-value of 0.000 is 0.05 or less, the decision is to reject H_0 .

Therefore, these results do provide enough evidence to establish that the mean right foot length of men is longer than the mean right foot length of women.

When 10 men and 10 women were randomly selected, the length of their right foot was measured in centimeters from the back of their heel to the tip of their toe. The results are summarized below.

Sample		Length (in cm) of Right Foot								
Men	25.6	27.9	28.2	27.5	27.0	26.8	29.1	28.6	25.8	27.4
Women	24.5	22.3	25.2	24.3	25.8	26.1	25.4	25.7	24.3	24.6

Estimate the actual difference in the mean right foot length of men and women? Construct the confidence interval estimate under the assumption that the right foot length of both men and women reasonably follow a normal distribution. Express the estimate both symbolically and verbally.

L1	L2	L3 3
25,6 27,2 28,5 27,2 26,1 29,1	<u> </u>	
L3(1)=		

$$1.52 < \mu_1 - \mu_2 < 3.62$$

The actual difference in the mean right foot length of men and women is estimated to be between 1.52 cm and 3.62 cm with 95% confidence.

Therefore, the mean right foot length of men is anywhere from 1.52 cm to 3.62 cm longer than the mean right foot length of women.

A random sample of 10 people had the length of both their left foot and right foot measured in centimeters from the back of their heel to the tip of their toe. The results are displayed below.

	Length of Foot (in cm)									
Left Foot	24.1	23.5	28.6	27.8	26.9	25.8	26.6	25.1	29.3	25.3
Right Foot	23.9	23.2	28.8	27.8	27.0	25.5	26.7	24.9	29.7	25.0
Difference	0.2	0.3	- 0.2	0	- 0.1	0.3	- 0.1	0.2	- 0.4	0.3

Is there a significant difference in the length of a person's left foot and right foot on the average? Use α = 0.01 and assume that the length of a person's left foot and right foot both reasonably follow a normal distribution.

This data involves two dependent samples.

$$H_0: \mu_d = 0$$

$$H_1: \mu_d \neq 0$$

Use
$$\alpha = 0.01$$

L1	L2	鄆	3						
24.1 23.5 28.6 27.8 26.9 26.6	23.9 23.8 23.8 27.7 25.7 22.2 22.2 22.2 22.2 22.2 22.2	.2 3.2 0 1.1 3.1							
L3 =L1	L3 =L1-L2								

p-value
$$\approx 0.537$$

Since the p-value of 0.537 is not 0.01 or less, the decision is to not reject H_0 .

Therefore, at the 0.01 level of significance, there is no significant difference in the length of a person's left foot and right foot on the average.

A sample of 8 people participated in a low calorie weight loss program for 16 weeks. Each of the participants had their weight (in pounds) recorded before they started the program and after they finished the program. The following results were obtained.

Participant	1	2	3	4	5	6	7	8
Weight Before	187	243	168	203	159	215	230	196
Weight After	172	222	151	175	146	207	206	177
Difference	15	21	17	28	13	8	24	19

At the 0.05 level of significance, test the claim that, on average, this 16 week low calorie weight loss program is effective? Assume that the amount of weight people lose after 16 weeks participating in this low calorie weight loss program reasonably follows a normal distribution.

This data involves two dependent samples.

$$H_0: \mu_d \le 0$$

$$H_1: \mu_d > 0$$

Use
$$\alpha = 0.05$$

L1	L2	鄆	3					
187 187 168 169 151 150 150 150 150 150 150 150 150 150	12215 12215 1245 1206 1206	1517 1217 1218 1218 1218 1218 1218 1218 12						
L3 =L1-L2								

p-value
$$\approx 4.2E - 5 \approx 0.000$$

Since the p-value of 0.000 is 0.05 or less, the decision is to reject H_0 .

Therefore, at the 0.05 level of significance, the claim that, on average, this 16 week low calorie weight loss program is effective is valid.

A sample of 8 people participated in a low calorie weight loss program for 16 weeks. Each of the participants had their weight (in pounds) recorded before they started the program and after they finished the program. The following results were obtained.

Participant	1	2	3	4	5	6	7	8
Weight Before	187	243	168	203	159	215	230	196
Weight After	172	222	151	175	146	207	206	177
Difference	15	21	17	28	13	8	24	19

Estimate the mean amount of weight a person participating in this low calorie weight loss program can expect to lose at the end of 16 weeks. Assume that the amount of weight people lose after 16 weeks participating in this low calorie weight loss program reasonably follows a normal distribution. Express the estimate both symbolically and verbally.

This data involves two dependent samples.

L1	L2	鄆	3			
187 243 168 203 159 215 230	172 151 151 175 146 206	151783 121783 182				
L3 =L1-L2						

$$12.8 < \mu_d < 23.4$$

The mean amount of weight a person participating in this low calorie weight loss program can expect to lose at the end of 16 weeks is estimated to be between 12.8 pounds and 23.4 pounds.

The instructor of a CPR (cardiopulmonary resuscitation) training course administered a test designed to measure a person's understanding of the CPR procedure to the students both at the beginning and at the conclusion of the course. The student's post-training and pre-training CPR test scores are provided in the table below.

Student	1	2	3	4	5	6	7	8
Post-Training	19	21	16	23	24	21	25	22
Pre-Training	6	9	2	11	6	11	18	7
Difference	13	12	14	12	18	10	7	15

Use a 99% confidence interval to estimate the mean improvement in the CPR test score of students upon completion of this training course. Assume that the differences in the post-training and pre-training CPR test scores are normally distributed. Express the estimate both symbolically and verbally.

This data involves two dependent samples.

L1	L2	1 83	3				
19 21 16 23 24 21 25	6 9 11 6 11 18	132 142 18 10 7					
L3 =L1-L2							

$$8.6 < \mu_d < 16.7$$

The mean improvement in the CPR test score of students upon completion of this training course is estimated to be between 8.6 and 16.7 with 99% confidence.