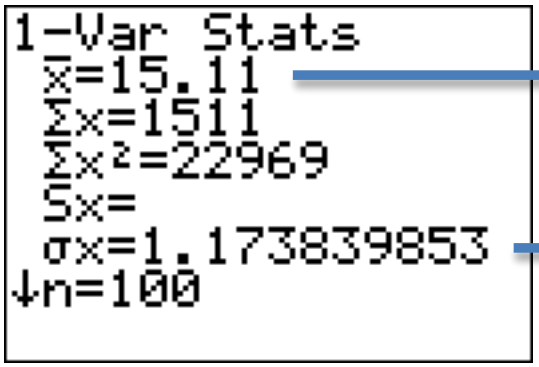
You are allowed to operate a calculator and refer to one page (front and back of standard 8.5 by 11 inch sheet) of notes while taking this examination. Your solutions should be clear, complete, and sufficiently detailed in order to demonstrate your understanding and communicate your reasoning and method of solving the problem. Each problem will be evaluated on a 5-point standard rubric.

Student's Name

Problem 1

The number of commercials (X) shown during a single hour of children's television programming is approximated by the following probability distribution.

X	p_X
13	10.4%
14	19.7%
15	31.5%
16	25.3%
17	13.1%

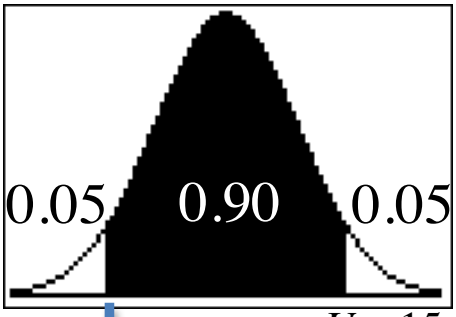


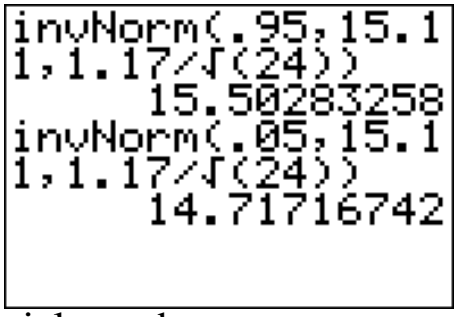
$\mu_X \approx 15.11$

$\sigma_X \approx 1.17$

For a random sample of 24 one hour blocks of children's television programming, find the narrowest interval that would accurately predict the mean number of commercials shown during a single hour of children's television programming 90% of the time.

The results of the Central Limit Theorem can be applied since this problem involves the mean for a random sample of size n = 24.





$U \approx 15.5$ commercials per hour

$L \approx 14.7$ commercials per hour

For a random sample of 24 one hour blocks of children's television programming, the narrowest interval that would accurately predict the mean number of commercials shown during a single hour of children's television programming 90% of the time ranges from a low of 14.7 to a high of 15.5 commercials per hour.

Problem 2

The results of a nationwide Gallup poll concerning a person's race and their opinion on the death penalty are summarized in the table below.

		Person's Race			
		White	Black	Hispanic	
Person's Opinion on the Death Penalty	In Favor	925	277	394	1596
	Opposed	362	415	317	1094
		1287	692	711	2690

(a) What percentage of the people polled were opposed to the death penalty?

$$p_{\text{Opposed}} = \frac{362 + 415 + 317}{2690} = \frac{1094}{2690} \cdot 100\% \approx 40.7\%$$

(b) What percentage of the people polled were either in favor of the death penalty or white?

$$p_{Favor \text{ or White}} = \frac{925 + 277 + 394 + 362}{2690} = \frac{1958}{2690} \cdot 100\% \approx 72.8\%$$

(c) What percentage of the people polled were Hispanic and opposed to the death penalty?

$$p_{Hispanic \text{ and Opposed}} = \frac{317}{2690} \cdot 100\% \approx 11.8\%$$

Problem 3

Refer to the results of the Gallup poll presented in Problem 2 to respond to the following questions.

(a) What proportion of the people polled was in favor of the death penalty given that they were black?

$$p_{\text{Favor given Black}} = \frac{277}{277+415}$$
$$= \frac{277}{692} \approx 0.400 = 40.0\%$$

(b) Based on these results, is a person's opinion on the death penalty independent of their race?

No. Since

$$p_{\text{Favor given Black}} \neq p_{Favor}$$

$$\frac{277}{692} \approx 40.0\% \neq 59.3\% \approx \frac{1596}{2690} = \frac{925+277+394}{2690}$$

these results indicate that a person's opinion on the death penalty is not independent of their race. It appears that a person's race does affect their opinion on the death penalty.

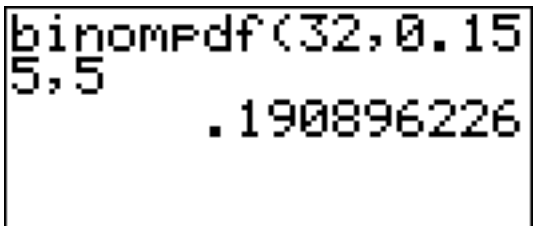
Problem 4

According to a recent Gallup poll, about 15.5% of adults worry about their weight all of the time.

(a) For a randomly selected class containing 32 college students, what is the likelihood that exactly five of these students worry about their weight all of the time?

The binomial probability distribution with n = 32, p = 0.155, and x = 5 can be used since an adult either worries about their weight all of the time or they do not (two possible outcomes).

$$\begin{aligned} p_{\text{exactly five}} &= p_{X=5} \\ &= p_5 \\ &\approx 0.191 \end{aligned}$$



For a randomly selected class containing 32 college students, the likelihood that exactly five of these students worry about their weight all of the time is about 19.1%.

(b) How many students in a randomly selected class containing 32 college students are expected to worry about their weight all of the time?

The binomial probability distribution with n = 32 and p = 0.155 can be used since an adult either worries about their weight all of the time or they do not (two possible outcomes).

$$E(X) = \mu_X = n \cdot p = 32 \cdot 0.155 = 4.96$$

Around 5 students in a randomly selected class containing 32 college students are expected to worry about their weight all of the time.

Problem 5

According to a recent Gallup poll, about 15.5% of adults worry about their weight all of the time. What is the probability that the number of students who worry about their weight all of the time in a randomly selected class containing 32 college students falls within two standard deviations of the mean?

The binomial probability distribution with n = 32 and p = 0.155 can be used since an adult either worries about their weight all of the time or they do not (two possible outcomes).

$$\mu_X = n \cdot p = 32 \cdot 0.155 = 4.96$$

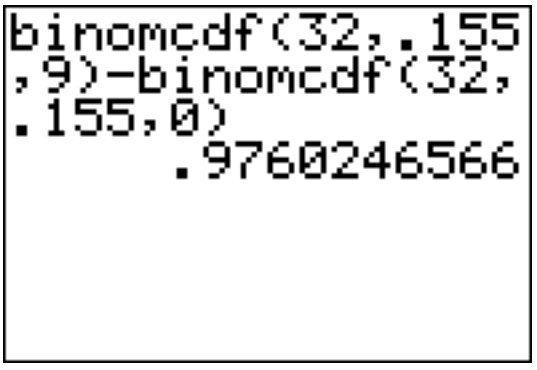
$$\sigma_X = \sqrt{n \cdot p \cdot (1-p)} = \sqrt{32 \cdot 0.155 \cdot (1-0.155)} \approx 2.05$$

$$\mu_X - 2 \cdot \sigma_X < X < \mu_X + 2 \cdot \sigma_X$$

$$4.96 - 2 \cdot 2.05 < X < 4.96 + 2 \cdot 2.05$$

$$0.86 < X < 9.06$$

$$p_{\text{within two standard deviations of mean}}$$
$$= p_{1 \le X \le 9}$$
$$= p_{X \le 9} - p_{X \le 0}$$
$$\approx 0.976$$

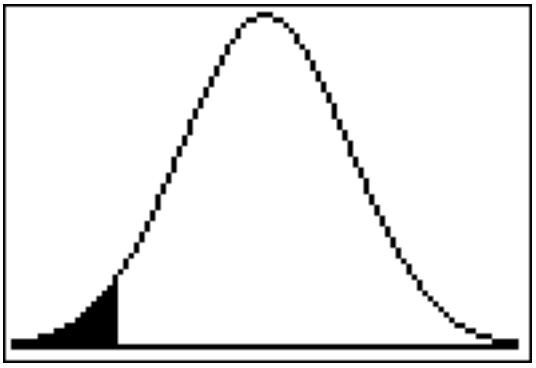


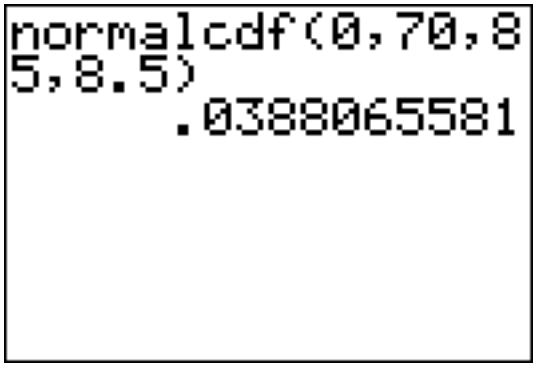
Thus, the probability that the number of students who worry about their weight all of the time in a randomly selected class containing 32 college students falls within two standard deviations of the mean is 97.6%.

Problem 6

After an 8 hour fast, a person's blood glucose level (in milligrams of glucose per deciliter of blood) is normally distributed with a mean of 85 mg/dl and a standard deviation of 8.5 mg/dl.

(a) Hypoglycemia (or low blood sugar) is defined as a blood glucose level below 70 mg/dl after an 8 hour fast. According to this definition, what proportion of people in the population has hypoglycemia?

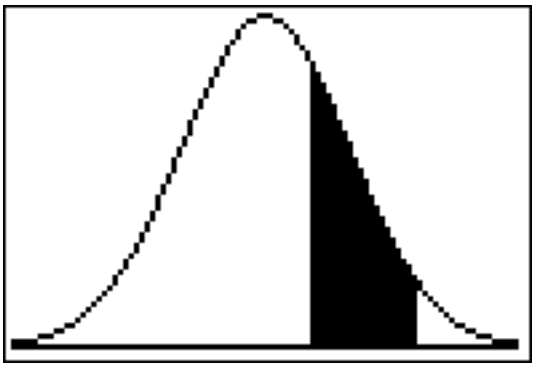


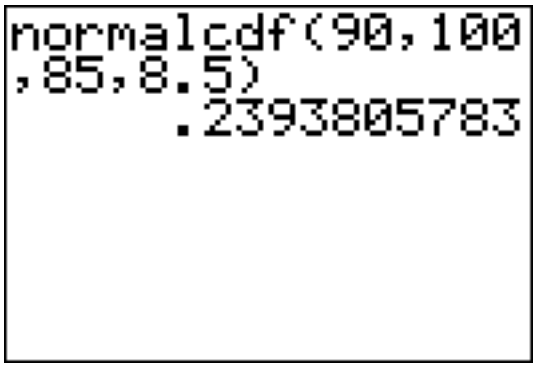


$$p_{\text{below } 70} = p_{\text{X<70}} \approx 0.039$$

According to this definition, about 3.9% of the people in the population has hypoglycemia (or low blood sugar).

(b) Would it be unusual for a person to have a blood glucose level between 90 and 100 mg/dl after an 8 hour fast?





$$p_{\text{between 90 and 100}} = p_{\text{90<X<100}} \approx 23.9\%$$

No. It would not be unusual for a person to have a blood glucose level between 90 and 100 mg/dl after an 8 hour fast since the probability that this would occur 23.9% is not 5% or less.