

Quiz 2 Solution

June 17 2019

$$\int \frac{x^7 - 23x^5 + 62x^4 - 82x^3 + 101x^2 - 68x + 10}{x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24} dx$$

Solution:

$$\begin{aligned} & \int \frac{x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24}{x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24} \\ & + \frac{7x^6 - 42x^5 + 95x^4 - 132x^3 + 150x^2 - 100x + 32}{x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24} \\ & + \frac{x^2 + 2}{x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24} dx \end{aligned}$$

$$\begin{aligned} & 1 + \ln |x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24| \\ & + \int \frac{x^2 + 2}{x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24} dx \end{aligned}$$

Factor the bottom:

$$\begin{array}{r|cccccccc} 2 & 1 & -7 & 19 & -33 & 50 & -50 & 32 & -24 \\ & & 2 & -10 & 18 & -30 & 40 & -10 & 24 \end{array}$$

$$\begin{array}{cccccccc|c} 1 & -5 & 9 & -15 & 20 & -10 & 12 & & 0 \\ (x-2)(x^6 - 5x^5 + 9x^4 - 15x^3 + 20x^2 - 12x + 12) \end{array}$$

$$\begin{array}{r|ccccccc} 2 & 1 & -5 & 9 & -15 & 20 & -10 & 12 \\ & & 2 & -6 & 6 & -18 & 4 & -12 \end{array}$$

$$\begin{array}{cccccc|c} 1 & -3 & 3 & -9 & 2 & -6 & & 0 \\ (x-2)^2(x^5 - 3x^4 + 3x^3 - 9x^2 + 2x - 6) \end{array}$$

$$\begin{array}{c|cccccc} 3 & 1 & -3 & 3 & -9 & 2 & -6 \\ & & 3 & 0 & 9 & 0 & 6 \\ \hline \end{array}$$

$$\begin{array}{cccccc|c} 1 & 0 & 3 & 0 & 2 & & 0 \\ \hline \end{array}$$

$$(x-2)^2(x-3)(x^4+3x^2+2)$$

$$(x-2)^2(x-3)(x^2+1)(x^2+2)$$

Now use pfd on:

$$\frac{x^2+2}{(x-2)^2(x-3)(x^2+1)(x^2+2)}$$

or

$$\frac{1}{(x-2)^2(x-3)(x^2+1)}$$

Then:

$$\frac{1}{(x-2)^2(x-3)(x^2+1)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x-3} + \frac{Dx+E}{x^2+1}$$

$$1 = A(x-2)(x-3)(x^2+1) + B(x-3)(x^2+1) + C(x-2)^2(x^2+1) + (Dx+E)(x-2)^2(x-3)$$

Let x=2:

$$1 = B(-1)(5) \Rightarrow B = -\frac{1}{5}$$

Let x=3:

$$1 = C(1)(10) \Rightarrow C = \frac{1}{10}$$

Equate:

$$x^4 : 0 = A + C + D \Rightarrow -\frac{1}{10} = A + D$$

$$x^3 : 1 = 6A - 3B + 2C - 12E \Rightarrow 0 = 6A - 12E \Rightarrow 0 = A - 2E$$

$$x^2 : 0 = -5A + B - 4C - 12D + 16E \Rightarrow \frac{3}{5} = -5A - 12D + 16E$$

$$\text{solving the system: } A = -\frac{1}{25}, D = -\frac{3}{50}, E = -\frac{1}{50}$$

So now we have

$$\frac{-\frac{1}{25}}{x-2} + \frac{-\frac{1}{5}}{(x-2)^2} + \frac{\frac{1}{10}}{x-3} + \frac{-\frac{3}{50}x - \frac{1}{50}}{x^2+1}$$

Now lets evaluate:

$$\int \frac{-\frac{1}{25}}{x-2} + \frac{-\frac{1}{5}}{(x-2)^2} + \frac{\frac{1}{10}}{x-3} + \frac{-\frac{3}{50}x - \frac{1}{50}}{x^2+1} dx$$
$$-\frac{1}{25} \ln|x+2| + \frac{1}{5(x-2)} + \frac{1}{10} \ln|x+3| - \frac{3}{100} \ln|x^2+1| - \frac{1}{50} \tan^{-1}(x) + C$$

Putting this together with what we integrated so far:

$$1 + \ln|x^7 - 7x^6 + 19x^5 - 33x^4 + 50x^3 - 50x^2 + 32x - 24|$$
$$-\frac{1}{25} \ln|x+2| + \frac{1}{5(x-2)} + \frac{1}{10} \ln|x+3| - \frac{3}{100} \ln|x^2+1| - \frac{1}{50} \tan^{-1}(x) + C$$

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