Math 31 More Applications to First Order Differential Equations July 9, 2019

1. The rate at which a population grows is proportional to the square root of the number of people. If $P(0) = 100$ and $P(1) = 121$, then find a model for this population.

2. A culture of bacteria is subjected to a controlled environment in order to test a new growth inhibitor. Empirical data obtained from these tests provides a growth model for this bacteria in this particular environment suggesting that the rate of growth is proportional to the square root of the amount of bacteria present at time *t*. If 80 grams of this bacteria is placed in this special environment grows to 125 grams in 1 hour, then write an appropriate differential equation and solve it.

3. The rate at which the population of social anarchists grows in the totalitarian regime of *Paraboland* is inversely proportional to the square of the population. If *Paraboland* initially experiences 10 such anarchists, and it will take 3 months for this population to double, then find the population of these anarchists as a function of time.

4. The major water reservoir in the Republic of Flatland holds 100 million gallons supplies the city of Trigland with 1 million gallons a day. The reservoir is partly refilled by a spring that provides 0.9 million gallons a day and the rest, 0.1 million gallons a day, by run-off from the surrounding land. The spring is clean, but the run-off contains salt with a concentration of 0.0001 pound per gallon. Assume that there is no salt in the reservoir initially and that the reservoir is well mixed. Find the concentration of salt in this reservoir as a function of time.

5. Suppose that this reservoir whose capacity is 100 million gallons, but is initially only half full of water, supplies this city with only 0.5 million gallons a day. Suppose also that it is partly refilled by a spring that provides 0.9 million gallons a day and an additional 0.1 million gallons a day is provided to the reservoir by run-off from the surrounding land. The spring is still clean and the run-off still contains salt with a concentration of 0.0001 pound per gallon. Assume that there is initially 200 pounds of salt in the reservoir and that the all-familiar mathematical god keeps the reservoir well mixed with his metaphoric spoon. Find the concentration of salt in this reservoir as a function of time.

6. ~~Two tanks, each containing 10 L of solution consisting of a pollutant dissolved in water. A solution containing 3 g/L of pollution flows into tank 1 at a rate of 4 L/min and the solution in tank 2 flows out at the same rate. In addition, solution flows into tank 1 from tank 2 at a rate of 1 L/min, and solution flows in the opposite direction at a rate of 5 L/min. Initially, tank 1 contains 50 g of solution whereas tank 2 contains 10 g of this solution. Find the amount of this pollutant in each tank at time *t*.~~

7. Newton's Law of Cooling states that $\frac{dT}{dt} = k(T - T_m)$ where $T(0) = T_0$

   i) Solve this differential equation to find a function for *T* in terms of *t*.

ii) Suppose that a 250° ham is removed from the oven and placed on the counter top of a kitchen whose temperature is 85°. If this ham cools down to 220° in 8 minutes, then how long will it take for this ham to cool down to 180°?

8. A population of 500 *Lagrangian toads* grows to 800 in 5 years, then to 1000 in 10 years. Assuming this growth is in accordance with our logistics model, then what is our population after *t* years? When will this population explode to 2000?

9. A population of 30 *Pythagorean Sabertooths* grows to 40 in 3 months, then to 45 in 6 months. Assuming this growth is in accordance with our logistics model, then what is our population after *t* years? When will this population reach 70?

10. Consider my cat *Pythagoras* with his pool and hoses. This time, one hose supplies the pool with 2 gallons per minute and has liquid catnip at a concentration of 0.2 grams per gallon. His second hose supplies my pool with a clean supply of water at a rate of 3 gallons per minute. Unfortunately, the pool has a leak and dumps water out at a rate of 4 gallons per minute. If the pool has a capacity of 500 gallons, is initially only half full, and initially contains no liquid catnip, then how much liquid catnip is in my pool after *t* minutes during the time it is reaching its capacity?

11. Consider my other cat *Jolie* with her pool and hoses. One hose supplies the pool with 4 gallons per minute and has liquid catnip at a concentration of 2 grams per gallon. Her second hose supplies my pool with a clean supply of water at a rate of 3 gallons per minute. This pool, too, has a leak and dumps water out at a rate of 5 gallons per minute. If the pool has a capacity of 1000 gallons, is initially only half full, and initially contains no liquid catnip, then how much liquid catnip is in my pool after *t* minutes during the time it is reaching its capacity?

12. Solve the following differential equations:

i) $$y' + \frac{3y}{x^2 - x - 2} = \frac{1}{x^4 - x^2 - 4x - 4}$$

ii) $$\left(2x^4 + x^3 - 2x^2 + 2x - 12\right)\frac{dy}{dx} = \left(2x^5 - 3x^4 + 3x^3 + 10x^2 - 17x - 22\right)y\sqrt{\ln^2 y + \ln y + 1}$$