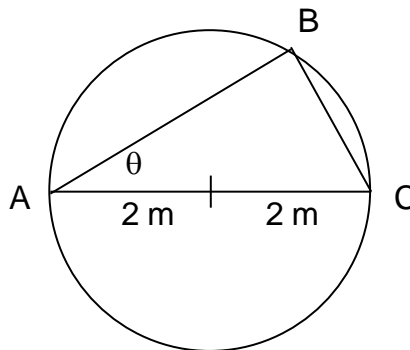


1. A 10 cm piece of wire is cut into two pieces. One piece is bent into the shape of a square while the other piece is bent into a circle. How should this wire be cut in order to minimize the total area of these two figures? How should this wire be cut in order to maximize the total area?
2. If a fish is swimming at a speed  $v$  relative to the water, the energy expended is proportional to  $v^3$ . It is believed that migrating fish naturally attempt to minimize the total energy required to swim a fixed distance. If fish are swimming against a current  $u$ , ( $u < v$ ), then the time required to swim a distance  $L$  is  $L/(v - u)$  and the total energy  $E$  required to swim this distance is given by

$$E(v) = av^3 \cdot \frac{L}{v - u}$$

where  $a$  is the constant of proportionality. Determine the value of  $v$  that minimizes  $E$  and sketch the graph of  $E$ .

3. I wish to travel from point A to point C which is diametrically opposite from point A across a circular lake whose radius is 2 miles. If I can swim at 2 miles per hour but walk at 4 miles per hour, then at what angle  $\theta$  should I depart to minimize the amount of time to get from point A to point C?



4. A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the dimensions that will produce the largest area for the window.

5. At what point on the line, whose equation is  $y = 2x - 1$ , is the distance to the point  $(1, -3)$  the shortest?
6. Consider both the graph of the parabola, whose equation is  $y = 5 - x^2$ , and a rectangle above the  $x$ -axis with one side on the  $x$ -axis and two vertices on the parabola. Where are the vertices of such a rectangle with largest area?
7. If the amount of a particular drug in a patient's blood after  $t$  hours is given by  $f(t) = \frac{t}{t^2 + 9}$ , when will the drug concentration be the greatest?
8. When one coughs, one uses a high-speed stream of air to clear your trachea, or windpipe. During the cough, the trachea contracts, forcing the air to move faster. But this also increases the friction. If the trachea contracts from a normal radius of 3 cm to a radius of  $r$  cm, then the velocity of the airstream is given by  $V(r) = k(3 - r)r^2$  where the constant  $k$  is determined by the length and elasticity of the trachea. Determine the radius  $r$  that maximizes this velocity.