1. Consider the following five sets. Complete the adjoining columns by determining for each set how many elements and how many subsets are associated with each. Note: given any set, the empty set (i.e., the set that contains nothing) and the original set itself are subsets of it.

Set:	**# of elements:**	**# of subsets:**
$\{\ \}$		
$\{a\}$		
$\{a,b\}$		
$\{a,b,c\}$		
$\{a,b,c,d\}$		

Determine how many subsets one can form from a set of *n* elements. Can you verify this with $n=5$ (i.e., list these sets)?

2. A survey of 120 people was conducted to determine the numbers who watched the three major television networks *ABC*, *CBS*, and *NBC*. The results of this survey are provided below:

Networks:	**# of people:**
ABC	55
CBS	40
NBC	30
ABC and *CBS*	10
ABC and *NBC*	12
CBS and *NBC*	8
ABC, *CBS* and *NBC*	5

How many people did not watch any of these networks? How many did not watch *ABC*? How many people watched *ABC* and *CBS* but not *NBC*?

3. If 52 students are taking a mathematics course, 38 are taking an English course, and 12 are taking both a mathematics course and an English course, then how many are taking either a mathematics course or an English course? Count this number two ways: first, with a Venn diagram and second, with the addition property.

4. There are 15,000 students currently attending *Flatland State University*. Through careful surveys, it is determined that 3500 of these students are special needs students. It is also determined that 2800 of the students attending this institution are currently enrolled in evening courses. If 5500 students are either special needs students or attending evening classes, then determine the following by using a Venn diagram representation of the problem situation:

   i) The number of students who are *both* attending evening courses and are special needs students.

   ii) The number of students currently enrolled who are not special needs students.

   iii) The number of special needs students who are not taking evening courses.

   iv) The number of students who are both not special needs students and not enrolled in evening courses.

5. I have an unreasonably large collection of 15 cats roaming my home.

   8 are tailless
   6 are afraid of mice
   3 have Greek names
   5 are both tailless and afraid of mice
   3 are tailless and have Greek names
   2 are afraid of mice and have Greek names
   2 are tailless, afraid of mice, and have Greek names

   i) How man of my cats are tailless, afraid of mice, but do not have Greek names?

   ii) How man are tailless, afraid of mice, or have Greek names?

   iii) How many have tails, fear no mice, and do not have Greek names?

6. Use a *Venn Diagram* model to represent the following universe and sets, then answer the questions that follow. Note that I did not indicate how many students satisfy all three conditions. You might need to consider more than one option/scenario by making some assumptions about how many students satisfy all three conditions.

18 students are surveyed.
10 students are mathematically literate.
4 are highly knowledgeable of world religions.
9 can speak at least three languages fluently.
3 are mathematically literate and are highly knowledgeable of world religions.
5 are mathematically literate and can speak at least three languages fluently.
2 are highly knowledgeable of world religions and can speak at least three languages.

i) How many of these students are knowledgeable of world religions but are neither mathematically literate nor can speak at least three languages?

ii) How many of these 18 students are mathematically literate, or highly knowledgeable of world religions, or can speak at least three languages?

iii) How many of these 18 students are not mathematically literate, nor highly knowledgeable of world religions, nor can speak at least three languages?

7. i) Prove pictorially (i.e., with a Venn diagram) one of the *DeMorgan's Laws*.

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

ii) Illustrate this version of *DeMorgan's Law* with the following universe and sets:

$$U = \{a, b, c, d, e, f, g, h\}$$
$$A = \{a, b, c\}, B = \{b, c, d, e, f\}$$

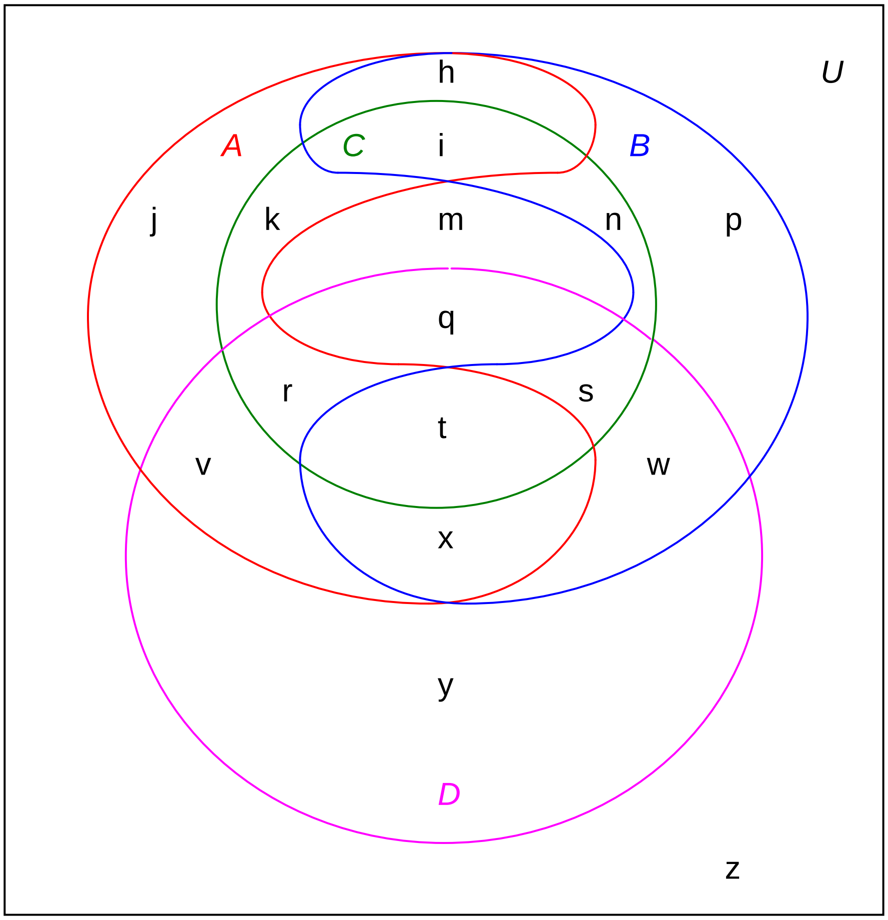
8. Consider next the union and intersection of more than two sets.

<u>Def</u>: $\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_n$, and $\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots \cap A_n$

Example: If $A_1 = \{1,2,5\}$, $A_2 = \{2,5,7\}$, and $A_3 = \{5,8\}$, then

$$\bigcup_{i=1}^{3} A_i = A_1 \cup A_2 \cup A_3 = \{1,2,5,7,8\} \text{ and } \bigcap_{i=1}^{3} A_i = A_1 \cap A_2 \cap A_3 = \{5\}$$

The following universe U is subdivided into 16 regions.



Determine each of the following:

i) $A \cap B \cap C \cap D$ ii) $(A \cap B) \cup (C \cap D)$ iii) $(\overline{A} \cap \overline{B}) \cap (C \cup D)$

9. Let $A_i = \{1, 2, 3, \ldots, i\}$. Determine each of the following:

i) $\bigcup_{i=1}^{5} A_i$ ii) $\bigcap_{i=1}^{5} A_i$

iii) $\bigcup_{i=1}^{n} A_i$ iv) $\bigcap_{i=1}^{n} A_i$

10. Let $A_i = \{i, i+1, i+2, i+3, \ldots\}$. Determine each of the following:

i) $\bigcup_{i=1}^{n} A_i$ ii) $\bigcap_{i=1}^{n} A_i$