1.   Provide a counter example for each of the following (let $x, y \in \mathbb{R}$):

     i) $\forall x \exists y \left(x = \dfrac{1}{y} \right)$

     ii) $\forall x \exists y \left(y^2 - x < 10 \right)$

     iii) $\forall x \forall y \left(x^2 \neq y^3 \right)$

     iv) $\forall x \forall y \left(x^2 = y^2 \rightarrow x = y \right)$

     v) $\forall x \exists y \left(y^2 = x \right)$

     vi) $\forall x \forall y \left(xy \geq x \right)$

2.   Let $R(x,y)$ be the statement "person x reads book y", and the universe of discourse for x and y be all Sierra College students and all Discrete Math books, respectively. Translate each of the following symbolically:

     i)   Every Sierra College student has read a Discrete Math book.

     ii)   A sierra College student has read all Discrete Math books.

     iii)   No Sierra College student has read all Discrete Math books.

     iv)   Every Sierra College student who has read a Discrete Math book will either be tired or will be happy (let $T(x)$ be the sentence "x will be tired" and $H(x)$ be the sentence x will be happy).

3.   Let $M(x,y)$ mean x admires y. Translate each of the following into words (write in a manner which is most likely stated in public):

     i) $\exists x \exists y \left((x \neq y) \wedge \left(M(l, x) \wedge M(l, y) \right) \right)$

ii) $\exists x \exists y \forall z \big((x \neq y) \wedge (M(x,z) \rightarrow M(y,z)) \big)$

iii) $\forall x \exists y \big(M(x,y) \wedge \forall z ((z \neq y) \rightarrow \neg M(x,z)) \big)$

4.   Let $K(x,y)$ be the sentence "x knows y". Allowing the universe of discourse be all persons, translate each of the following symbolically:

i)   Anyone who knows *Archimedes* knows *Pythagoras*.

ii)   There is someone whom no one knows.

iii) Someone knows nobody.

iv)   Everyone loves only one other.