Verify each of the following mathematical statements using Mathematical Induction:

1. $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$ for all $n \in Z^+$

2. $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all $n \in Z^+$

3. $\frac{d}{dx}\left(x^n\right) = nx^{n-1} \quad \forall n \in N$

4. $1^4 + 2^4 + 3^4 + \cdots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \quad \forall n \in N$

5. $6 \cdot 7^n - 2 \cdot 3^n$ is divisible by 4.

6. $\sum_{i=1}^{n} i(i!) = (n+1)! - 1 \quad \forall n \in N$

7. $11^n - 5^n$ is divisible by 3 $\forall n \in N$

8. Let f_n be the n$^{\text{th}}$ Fibonacci number. $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} f_{n-1} & f_n \\ f_n & f_{n+1} \end{bmatrix} \quad \forall n \in N$

9. $\sin x + \sin 3x + \sin 5x + \cdots + \sin(2n-1)x = \frac{1-\cos 2nx}{2\sin x} \quad \forall n \in N$

More Examples:

1. $2^{2n}-1$ is divisible by 3 $\forall n \in N$

2. $2n+1<2^n \quad \forall n \geq 3$

3. $2^n<(n+1)! \quad \forall n \geq 2$

4. 7^n-2^n is divisible by 5 $\forall n \in N$

5. $3^{2n}-1$ is divisible by 8 $\forall n \in N$

6. $n^2<n! \quad \forall n \geq 4$

7. Prove that the sum of the first *n* terms of a geometric series is given by the following two ways: directly and using *mathematical induction*.

$$S_n=\frac{a\left(1-r^n\right)}{1-r} \quad \forall n \in N$$