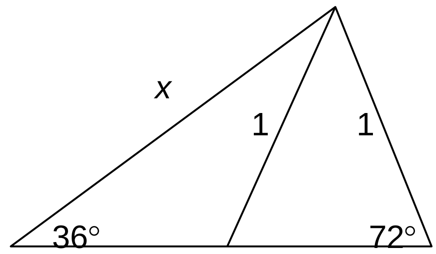
I. Find x, which is the ratio of the length of the long side of this triangle to the length of its short side (and be ready to be impressed with this value). Give both an exact value and an approximation.



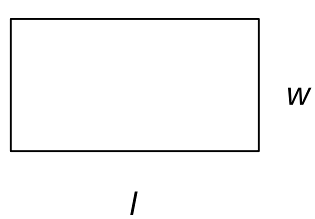
II. **<u>Def</u>:** The **Fibonacci** sequence is the sequence which is recursively defined as follows:

$$F_1 = 1,$$
$$F_2 = 1,$$
$$F_n = F_{n-1} + F_{n-2} \quad \text{where } n > 2$$

1. Given that $F_{23} = 28{,}657$ and $F_{25} = 75{,}025$, determine the value of F_{24}.

2. Given that $a = \frac{1+\sqrt{5}}{2}$ and $b = \frac{1-\sqrt{5}}{2}$, determine the exact value of $a+b$, ab, and $a^2 + b^2$

3. Consider the following rectangle with side lengths $l = 3+\sqrt{5}$ and $w = 1+\sqrt{5}$:



Is this a golden rectangle? Justify your answer.

4. Use your calculator to find $\frac{F_2}{F_1}, \frac{F_3}{F_2}, \frac{F_4}{F_3}, \frac{F_5}{F_4}, \frac{F_6}{F_5}, \frac{F_7}{F_6}, \frac{F_8}{F_7}$. What is $\lim_{n\to\infty} \frac{f_n}{f_{n-1}}$?

5. Let ϕ represent the golden ratio, which is the solution to the equation $x^2 = x+1$, meaning of course that $\phi^2 = \phi + 1$.

   Multiplying both sides of the equation $\phi^2 = \phi + 1$ by ϕ yields:

$$\begin{aligned} \phi^3 &= \phi^2 + \phi \\ &= \phi + 1 + \phi \\ &= 2\phi + 1 \end{aligned}$$

   Express ϕ^4, ϕ^5, ϕ^6, and ϕ^7 in terms of ϕ as well and describe any pattern that you observe.