Theorem 2.1.1 Logical Equivalences

Given any statement variables *p, q*, and *r* , a tautology **t** and a contradiction **c**, the following logical equivalences hold.

1.	*Commutative laws:* $p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2.	*Associative laws:* $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3.	*Distributive laws:* $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4.	*Identity laws:* $p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5.	*Negation laws:* $p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6.	*Double negative law:* $\sim(\sim p) \equiv p$	
7.	*Idempotent laws:* $p \wedge p \equiv p$	$p \vee p \equiv p$
8.	*Universal bound laws:* $p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9.	*De Morgan's laws:* $\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10.	*Absorption laws:* $p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11.	*Negations of* **t** *and* **c**: $\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$

Table 2.3.1 Valid Argument Forms (Rules of Inference)

Modus Ponens

$p \to q$
p
$\therefore q$

Elimination

a. $p \vee q$
$\sim q$
$\therefore p$

b. $p \vee q$
$\sim p$
$\therefore q$

Modus Tollens

$p \to q$
$\sim q$
$\therefore \sim p$

Transitivity

$p \to q$
$q \to r$
$\therefore p \to r$

Generalization

a. p
$\therefore p \vee q$

b. q
$\therefore p \vee q$

Proof by Division into Cases

$p \vee q$
$p \to r$
$q \to r$
$\therefore r$

Specialization

a. $p \wedge q$
$\therefore p$

b. $p \wedge q$
$\therefore q$

Conjunction

p
q
$\therefore p \wedge q$

Contradiction Rule

$\sim p \to \mathbf{c}$
$\therefore p$