1. Determine if $(G,*)$ is a group. If it is, identify the identity element and each element's inverse:

   i) $G=\{-1,1\}$ and $*$ is addition

   ii) $G=\{-1,1\}$ and $*$ is multiplication

   iii) $G=[-1,1]$ and $*$ is addition

   iv) $G=[-1,1]$ and $*$ is multiplication

   v) $G=\{-1,0,1\}$ and $*$ is addition

   vi) $G=\{-1,1,i,-i\}$ and $*$ is multiplication

   vii) $G=\{1\}$ and $*$ is multiplication

   viii) $G=\left\{\frac{a}{2^n}\right\}$ where $a\in\mathbb{Z}\setminus\{0\}$ and $n\in\mathbb{Z}$ and $*$ is multiplication

2. If $G=\mathbb{Z}$ and $a*b=a+b+1\ \forall a,b\in G$, then $(G,*)$ is a group. Verify that the four criteria for being a group are indeed satisfied.

3. Prove that $f(A\cup(B\cap C))\subseteq f(A)\cup[f(B)\cap f(C)]$

4. Is $f(A)\cup[f(B)\cap f(C)]\subseteq f(A\cup(B\cap C))$? If so, prove it. If not, provide a counter-example.