Recall that if we have a second order linear recurrence relation with constant coefficients of the form $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ with initial conditions a_0, a_1, where $r^2 - c_1 r - c_2 = 0$, then:

i) $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ is a solution to $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if $r_1 \neq r_2$ and

ii) $a_n = (\alpha_1 + n\alpha_2) r^n$ is a solution to $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if $r_1 = r_2$

1. Using the above, find an explicit formula for the following recurrence relations:

   i) $a_n = 3a_{n-1} + 10a_{n-2}$ where $a_0 = 2$ and $a_1 = 3$

   ii) $a_n = 4a_{n-1} + 21a_{n-2}$ where $a_0 = 1$ and $a_1 = 4$

   iii) $a_n = -8a_{n-1} - 7a_{n-2}$ where $a_0 = 3$ and $a_1 = 1$

   iv) $a_n = -11a_{n-1} - 30a_{n-2}$ where $a_0 = 2$ and $a_1 = 3$

   v) $a_n = -10a_{n-1} - 25a_{n-2}$ where $a_0 = 1$ and $a_1 = 5$

   vi) $a_n = 14a_{n-1} - 49a_{n-2}$ where $a_0 = 2$ and $a_1 = 1$

2. Use generating functions to answer the following questions:

   i) My cat *Pythagoras* is quite the spoiled cat. I always have three bulls of food placed out for him. The first bowl contains the smallest treats, consisting of 12 treats. They are so small, he takes three at a time when he eats them. The second bowl contains 10 slightly larger morsels that he takes two at a time. The third bowl contains 7 bite size items that he can take one at a time. In how many ways can he eat from these three bowls eight items?

   ii) What if *Pythagoras* broke into the factory that creates these three foods (and essentially has an unlimited supply of these items). In how many ways can he eat from these three food sources eight items?

   iii) I wish to post a letter with 1¢, 3¢, and 5¢ stamps (yes, this is 1942!). Consider a generating function for $\{a_r\}$ where a_r is the number of ways he can arrange exactly 3 of these stamps in a row with a total value of r cents.

a) in how many ways can his stamps total 8¢?

b) 4 stamps?

c) 3 or 4 stamps?