

Some definitions and computations:

- i) Experiment: an observation of a physical occurrence.
  - ii) Sample Space: the set of all possible outcomes of an experiment.
  - iii) Event: a subset of the sample space (a collection of outcomes of an experiment).
  - iv) Simple Event: an event consisting of only one element (i.e., a particular outcome of an experiment).
  - v) Impossible Event: an event that is the empty set (an event that cannot possibly occur).
  - vii) Theoretical Probability: the theoretical probability of an event occurring is  $p = \frac{m}{n}$  where the experiment has a sample space consisting of  $n$  elements with an event consisting of  $m$  elements.
  - viii)  $n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$  where  $n$  is an integer
  - ix)  $P(n, k) = \frac{n!}{(n-k)!}$  This is the means by which we can compute a permutation. Specifically, it represents the number of ways we can permute  $n$  items  $k$  at a time. Here, order **is** important.
  - x)  $\binom{n}{k} = C(n, k) = \frac{n!}{k!(n-k)!}$  This is the means by which we can compute a combination. Specifically, it represents the number of ways we can combine  $n$  items  $k$  at a time. In other words, this number represents the number of subsets of size  $k$  that can be created from a set of size  $n$ . Here, order is **not** important.
1. An arbitrarily chosen bag of M&M's consists of 24 green candies, 11 red candies, 20 blue candies, and 30 green candies. If one were to choose an arbitrary piece of candy from this bag, what is the probability that it would be red? Green? Grey? What is the probability that if three are chosen, they are of the same color?
  2. Use a four penny grid for the following experiment. What is the theoretical probability that a toss of the penny will result in the penny falling to a rest position without touching a line?

3. Consider the experiment of tossing a pair of dice. Determine the probability of showing two numbers whose sum is eight. Determine the probability that the difference in the numbers showing is no more than three. Provide the sample space of this experiment to answer this question.
4. Consider the following experiment: choosing a poker chip from a bag of chips consisting of 5 white chips, 10 red chips, and 15 blue chips. If one were to pull out one chip, observe its color, replace that chip back into the bag, and pick another chip and observe its color, what is the theoretical probability that:
  - i) the first is red and the second is blue?
  - ii) the first is blue if the second is white?
5. Consider the five card hand (selected from a standard deck of 52 cards):  $\{3\heartsuit, 7\heartsuit, 4\heartsuit, 9\spadesuit, K\clubsuit\}$ . What is the probability of randomly selecting 2 hearts from these five cards?
6. A manufacturer receives 60% of machine parts from supplier #1, and 40% from supplier #2. It is observed that 2% of parts from supplier #1 and 1% of parts from supplier #2 are defective. A part is selected at random and found to be defective. What is the probability that it came from supplier #1?
8. There is an abundance of colored markers in this room. Six are blue, seven are black, four are red, and three are green. What is the probability that an arbitrarily chosen collection of three pens will be of the same color? What is the probability that an arbitrarily chosen collection of three pens will be of distinct colors?
9. Given a standard deck of cards, what is the probability that one will be dealt:
  - i) a *four of a kind* hand?
  - ii) a *full house*?
10. A company receives a shipment of 12 computer modems, 4 of which are defective. In how many ways can one select 6 of these modems such that exactly 2 are defective? What is the probability that if one arbitrarily chooses 6 of these modems, exactly 2 are defective?
11. The state of *Algebrania* produces license plates consisting of four digits followed by three letters. How many different license plates can be produced? What is the probability that an arbitrarily chosen plate has letters from the word *Pythagoras* and whose digits are even if letters, but not digits, can be repeated?
12. The *Island of Misfit Variables* is inhabited by very superstitious rulers. It is a commonly held belief that no license plate shall hold an odd digit and no license plate shall possess the letters *m, a, t, h*. If each license plate consists of three of the acceptable digits, followed by four of the acceptable letters, then how many distinct plates are possible if we allow no repeat of any symbol?

13. How many distinct permutations of the letters in the word *possession* are there?
14. There is an abundance of colored markers in this room. Six are blue, seven are black, four are red, and three are green.
- i) In how many ways can I choose 3 pens?
  - ii) In how many ways can I choose three black pens?
  - iii) In how many ways can I choose three pens of the same color?
  - iv) What is the probability that if three are chosen, all are black?
  - v) What is the probability that if three are chosen, all are of the same color?
15. I am a student of a culinary school. There are ten of us enrolled in the program, and when we complete the program, only one of us will be selected to be honored with the official title of 'chef', while only two of us will make it to the position of 'assistant chef'. In how many ways can this be done?