Do the following subsets of the given set partition the larger set? If so, identify the equivalence classes. A partition of a set is a collection of disjoint subsets whose union is the original set.

1. The set is $\mathbb{N}$ and the subsets are the set of primes and the set of composite numbers.

2. The set is $\{0,1,2,3,4,5,6,7,8,9\}$ and the subsets are $\{0,1,2,3,7\},\{4,5\},\{6,8\},\{9\}$

3. The set is $\{0,1,2,3,4,5,6,7,8,9\}$ and the subsets are $\{0,1,2,3,7\},\{3,4,5\},\{6,8\},\{9\}$

4. The set is $\{0,1,2,3,4,5,6,7,8,9\}$ and the subsets are $\{0,1,2,7\},\{4,5\},\{6,8\},\{9\}$

5. The set of all students who started Math 15 here at Sierra College this semester is our set. Consider the subsets of those who will finish with a letter grade of 'C' or better and those who will finish the course with a letter grade below a 'C'.

6. The set is the integers and the subsets contain numbers whose distance from the origin is the same.

7. The set is $\mathbb{R}$ and the subsets are $\{A_i \mid i \in \mathbb{Z}\}$ where $A_i = (i-1, i]$

Create a partition on the following set and identify the equivalence classes. There is not necessarily one answer.

1. The set is $S = \left\{a+bi \mid a,b \in \mathbb{Z} \text{ and } |a|=|b| \text{ and } i=\sqrt{-1}\right\}$

2. The set is $\mathbb{R}$ and we wish to partition this set into countably infinite (i.e., the size of $\mathbb{Z}$) many subsets.

3. The set of all individuals in this classroom today.

Answers:

1. We are not forming a partition of $\mathbb{N}$ for the union of these two subsets is not all of $\mathbb{N}$, after all, the union of these two subsets does not contain 1 since 1 is neither prime nor composite.

2. This does form a partition and the equivalence classes are $[2],[5],[6],[9]$ (each of these numbers 2, 5, 6, and 9 are merely arbitrarily chosen from each of the subsets.

3. This does not form a partition for the first two sets are not disjoint, after all, they both contain 3.

4. This does not form a partition for 3 is not in the union of these subsets (therefore, the union of these subsets is not all of $\{0,1,2,3,4,5,6,7,8,9\}$).

5. This does not form a partition for there are some students who started this course this semester who are not finishing it and therefor will not be receiving a grade.

6. This does form a partition on $\mathbb{Z}$ where the disjoint subsets are $\{0\},\{-1,1\},\{-2,2\},\ldots.$ and appropriate equivalence classes might be $[0],[1],[2],\ldots.$

1. A natural partition contains the subsets: $\{0\},\{1+i,1-i,-1+i,-1-i\},\{2+2i,2-2i,-2+2i,-2-2i\},\ldots.$ whose partitions could perhaps be represented as $[0],[1],[2],\ldots.$

2. Perhaps $\mathbb{R}=\cdots\cup\lfloor -2\rfloor\cup\lfloor -1\rfloor\cup\lfloor 0\rfloor\cup\lfloor 1\rfloor\cup\lfloor 2\rfloor\cup\cdots$ will do.

3. We can partition in terms of age, truncated, or in terms of height, rounded to the nearest inch, or in terms of annual income, rounded to the nearest thousands of dollars, or….