Provide a clear and organized presentation. Show all of your work, provide exact values only, and completely simplify your answers.

1.   If a^2 is odd then $a+1$ is even

2. Prove that if a set has *n* elements, then its power set has 2^n elements $\forall n \in \mathbb{N}$ (hint: use induction for this).

3.　　Prove that $A \cup \overline{B} = \overline{B \setminus A}$

4. Prove that if the sum of the digits of a natural number is divisible by nine, then the natural number is divisible by nine.

5. A standard deck of *Lagrange Playing Cards* consists of 9 denominations and 5 suits. What is the probability of selecting five cards and drawing:

    i)   a three-of-a-kind?

    ii)   a full house (three cards of the same denomination and then the remaining two cards of the same denomination)?

    iii)   the first three of the same suit and the remaining two of distinct suits that are different from the first three?

    iv)   two pair?

6.   Among the 30,000 Sierra College students, 7% love math and among the 50,000 American River College students, 4% love math. What is the probability that an arbitrarily chosen student from these colleges is a Sierra College student if it is known that they love math?

7. Prove that if a fifth degree polynomial has integer coefficients, all of which are odd except the leading coefficient (it is even) and has a rational zero, then the zero must be expressible as the ratio of two integers, which in completely reduced form is expressed as the ratio of an odd integer to an even one.

8.   Determine both $\bigcup\limits_{i=1}^{n} A_i$ and $\bigcap\limits_{i=1}^{n} A_i$ if:

i) $A_i = \left[\dfrac{2}{i}, 3 + i\right)$

ii) $A_i = \{i, i + 1\}$