Provide a clear and organized presentation. Give exact values only and show all of your work.

1. Prove that $3x-2y$ is a factor of $(3x)^n-(2y)^n \quad \forall n\in\mathbb{N}$

2. Verify that the following argument is valid:

> Pigs can fly and sheep sing. If buffalo roam or birds chirp, then pigs can’t fly. If alligators snap, then bears climb. If crocodiles crawl, then bears climb. Alligators snap or crocodiles crawl. If both birds don’t chirp and bears climb, then the cycle of life continues. Therefore the cycle of life continues.

3. Prove that $\left(\cos\theta + i\sin\theta\right)^n = \cos n\theta + i\sin n\theta \quad \forall n \in \mathbb{N}$

4. Prove that $\sqrt{11} \notin \mathbb{Q}$

5. Among my nine cats is an elite subgroup of highly mathematically inclined cats. This subgroup is called the *Joint*. Let $J(x)$ represent that cat x is a member of the *Joint*. Determine who is in the *Joint*.

My collection of nine cats includes:

Pythagoras, Archimedes, Jolie, Theta, Cobalt, Pascal, Euler, Euclid, and Escher.

i) $\exists x \exists y \exists z \left(x \neq y \wedge x \neq z \wedge y \neq z \wedge J(x) \wedge J(y) \wedge J(z) \right)$

ii) $\sim \left(J(\text{Archimedes}) \wedge J(\text{Cobalt}) \right)$

iii) $\left(J(\text{Euler}) \vee J(\text{Theta}) \right) \rightarrow \forall x J(x)$

iv) $J(\text{Archimedes}) \rightarrow J(\text{Cobalt})$

v) $J(\text{Jolie}) \rightarrow J(\text{Euler})$

vi) $\left(J(\text{Pythagoras}) \vee J(\text{Pascal}) \right) \rightarrow \left(\sim J(\text{Escher}) \right)$

vii) $\left(J(\text{Pythagoras}) \vee J(\text{Cobalt}) \right) \rightarrow \left(\sim J(\text{Euclid}) \right)$

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology **t** and a contradiction **c**, the following logical equivalences hold.

1. *Commutative laws:* $p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:* $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:* $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:* $p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:* $p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:* $\sim(\sim p) \equiv p$	
7. *Idempotent laws:* $p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:* $p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:* $\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:* $p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* **t** *and* **c**: $\sim \mathbf{t} \equiv \mathbf{c}$	$\sim \mathbf{c} \equiv \mathbf{t}$
12. The 'Jeepers' Rule:	$p \to q \equiv \sim p \vee q$

Table 2.3.1 Valid Argument Forms (Rules of Inference)

Modus Ponens

$p \to q$
p
$\therefore\ q$

Elimination

a. $p \vee q$, $\sim q$, $\therefore\ p$

b. $p \vee q$, $\sim p$, $\therefore\ q$

Modus Tollens

$p \to q$
$\sim q$
$\therefore\ \sim p$

Transitivity

$p \to q$
$q \to r$
$\therefore\ p \to r$

Generalization

a. p, $\therefore\ p \vee q$

b. q, $\therefore\ p \vee q$

Proof by Division into Cases

$p \vee q$
$p \to r$
$q \to r$
$\therefore\ r$

Specialization

a. $p \wedge q$, $\therefore\ p$

b. $p \wedge q$, $\therefore\ q$

Conjunction

p
q
$\therefore\ p \wedge q$

Contradiction Rule

$\sim p \to \mathbf{c}$
$\therefore\ p$