Provide a clear and organized presentation. Show all of your work, completely simplify your answers, and give exact values only.

1. (10 pts) Prove that $B \cap \left(\bigcup_{i=1}^{n} A_i \right) = \bigcup_{i=1}^{n} \left(B \cap A_i \right)$ using our technique involving each side being a subset of the other.

2. (10 pts) Use mathematical induction to prove that $x - 2y$ is a factor of $x^n - 2^n y^n \quad \forall n \in \mathbb{N}$

3. (10 pts) Consider:

   i) Let $A_i = \{ i, i+3 \}$. Determine the following:

      a) $\bigcup_{i=1}^{n} A_i$

      b) $\bigcap_{i=1}^{n} A_i$

   ii) Let $A_i = \left[\frac{1}{i}, i+1 \right)$. Determine the following:

      a) $\bigcup_{i=1}^{n} A_i$

      b) $\bigcap_{i=1}^{n} A_i$

4. (10 pts) Translate each of the following using quantifiers ($L(x, y)$ means that person *x* loves person *y*):

   i) Everyone loves exactly two others (other than him/herself).

   ii) Whenever one is loved by someone, it is him/herself.