

Provide a clear and organized presentation. Show all of your work, completely simplify all answers, and give exact values only.

1. (10 pts) Let $f(x) = \ln(1 + x + \sqrt{1 + x^2})$ with $x > 0$. Show that f is 1-1.
2. (10 pts) Let $f(x) = \ln(1 + x + \sqrt{1 + x^2})$ with $x > 0$. Determine $f^{-1}(x)$.
3. (10 pts) Show that $(f(A) \cup f(B)) \cap (f(C) \cup f(D)) \subseteq f((A \cup B) \cap (C \cup D))$ if f is 1-1.
4. (20 pts) Consider the set $G = \mathbb{R} \setminus \{0, -e^2\}$ where $a * b = \frac{a}{b + e^2} \forall a, b \in G$.
 - i) If $(G, *)$ were a group, determine both the identity and inverse elements.
 - ii) Explain why $(G, *)$ is not a group.