

1. Determine if each of the following transformations T are linear transformations:

i) Let g be some nonzero continuous function defined over $[a, b]$. Define

$$T: C^0[a, b] \rightarrow R \text{ by } T(f) = \int_a^b \frac{f(x)}{g(x)} dx$$

ii) Let $T: P_2 \rightarrow P_2$ be defined by $T(a_0 + a_1x) = a_0 + (a_1 + 1)x$

2. Define $T : P_3 \rightarrow P_3$ by $T(a_0 + a_1x + a_2x^2) = (a_0 + a_2) + (2a_2 - a_0)x^2$. Determine each of the following:

i) $\text{Ker}(T)$

ii) $\dim(\text{Rng}(T))$

iii) $\text{Rng}(T)$

3. Let $T : P_3 \rightarrow C^0[a, b]$ be defined by $T(f) = f''$. Determine $\text{Ker}(T)$. Is T onto?

4. Let \bar{u} be a fixed vector in \mathbb{R}^3 . If $T : \mathbb{R}^3 \rightarrow \mathbb{R}$ is defined by $T(\bar{v}) = \langle \bar{u}, \bar{v} \rangle$. Describe in words what $\text{Ker}(T)$ represents.