Math 33 **Quiz 2** ***January 25, 2018***

Provide a clear and organized presentation. Show all of your work and provide exact values only. Completely simplify your answer.

$$y^3\left(x^3+3x^2-4\right)\frac{dy}{dx}=\left(y^2+\sqrt{y^2+3}\right)\left(x^4+5x^3+10x^2+x-6\right)$$

Separating variables, we have:

$$\int\frac{y^3}{y^2+\sqrt{y^2+3}}\,dy=\int\frac{x^4+5x^3+10x^2+x-6}{x^3+3x^2-4}\,dx$$

On the left-hand side, we will commit to the rationalizing substitution:

$$u=\sqrt{y^2+3},\quad u^2=y^2+3,\quad udu=ydy$$

And we will add our funky form of zero on the right-hand side. This give us:

$$\int\frac{u^2-3}{u^2+u-3}\,udu=\int\frac{x^4+3x^3-4x+2x^3+6x^2-8+4x^2+5x+2}{x^3+3x^2-4}\,dx$$

$$\int\frac{u^3-3u}{u^2+u-3}\,du=\int\left(x+2+\frac{4x^2+5x+2}{x^3+3x^2-4}\right)dx$$

And zero addition one more time yields:

$$\int\frac{u^3+u^2-3u-u^2-u+3+u-3}{u^2+u-3}\,du=\int\left(x+2+\frac{4x^2+5x+2}{x^3+3x^2-4}\right)dx$$

$$\int\left(u-1+\frac{u-3}{u^2+u-3}\right)du=\int\left(x+2+\frac{4x^2+5x+2}{x^3+3x^2-4}\right)dx$$

$$\int\left(u-1+\frac{u+\frac{1}{2}-\frac{7}{2}}{u^2+u-3}\right)du=\int\left(x+2+\frac{4x^2+5x+2}{x^3+3x^2-4}\right)dx$$

However, note that:

$$\begin{array}{c|cccc} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & 0 \end{array}$$

Giving us that $\frac{4x^2+5x+2}{x^3+3x^2-4}=\frac{4x^2+5x+2}{(x-1)(x+2)^2}$

$$\frac{4x^2+5x+2}{x^3+3x^2-4}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{(x+2)^2}$$

$$4x^2+5x+2=A(x+2)^2+B(x-1)(x+2)+C(x-1)$$

and if $x=1$, then $11=9A$, so $A=\frac{11}{9}$

and if $x=-2$, then $8=-3C$, so $C=-\frac{8}{3}$

and equating coefficients of x^2, we have that $A+B=4$, or that $B=\frac{25}{9}$.

So, $\int\left(u-1+\frac{u+\frac{1}{2}-\frac{7}{2}}{u^2+u-3}\right)du=\int\left(x+2+\frac{4x^2+5x+2}{x^3+3x^2-4}\right)dx$ becomes:

$$\int\left(u-1+\frac{u+\frac{1}{2}}{u^2+u-3}-\frac{7}{2}\cdot\frac{1}{\left(u+\frac{1}{2}\right)^2-\frac{13}{4}}\right)du=\int\left(x+2+\frac{11}{9}\cdot\frac{1}{x-1}+\frac{25}{9}\cdot\frac{1}{x+2}-\frac{8}{3}\cdot\frac{1}{(x+2)^2}\right)dx$$

$$\frac{1}{2}u^2-u+\frac{1}{2}\ln\left|u^2+u-3\right|-\frac{7}{2}\int\frac{1}{\frac{13}{4}\tan^2\theta}\cdot\frac{\sqrt{13}}{2}\sec\theta\tan\theta d\theta=\frac{1}{2}x^2+2x+\frac{11}{9}\ln\left|x-1\right|+\frac{25}{9}\ln\left|x+2\right|+\frac{8}{3}\cdot\frac{1}{x+2}+C$$

$$\frac{1}{2}\left(y^2+3\right)-\sqrt{y^2+3}+\frac{1}{2}\ln\left(y^2+\sqrt{y^2+3}\right)+\frac{7}{\sqrt{13}}\ln\left|\csc\theta+\cot\theta\right|=\frac{1}{2}x^2+2x+\frac{1}{9}\ln\left|x-1\right|^{11}\left|x+2\right|^{25}+\frac{8}{3}\cdot\frac{1}{x+2}+C$$

$$\frac{1}{2}y^2-\sqrt{y^2+3}+\frac{1}{2}\ln\left(y^2+\sqrt{y^2+3}\right)+\frac{7}{\sqrt{13}}\ln\left|\frac{2u+1+\sqrt{13}}{2\sqrt{u^2+u-3}}\right|=\frac{1}{2}x^2+2x+\frac{1}{9}\ln\left|x-1\right|^{11}\left|x+2\right|^{25}+\frac{8}{3}\cdot\frac{1}{x+2}+C_1$$

$$\frac{1}{2}y^2-\sqrt{y^2+3}+\frac{1}{2}\ln\left(y^2+\sqrt{y^2+3}\right)+\frac{7}{\sqrt{13}}\ln\left|\frac{2\sqrt{y^2+3}+1+\sqrt{13}}{2\sqrt{y^2+\sqrt{y^2+3}}}\right|=\frac{1}{2}x^2+2x+\frac{1}{9}\ln\left|x-1\right|^{11}\left|x+2\right|^{25}+\frac{8}{3}\cdot\frac{1}{x+2}+C_1$$