Let $A=\left\{1+x^2,1-x^2+x^4,2-x^2\right\}$ which spans a subspace of P_4. Determine an orthogonal basis for the same subspace that *A* spans using the *Gram-Schmidt* procedure assuming the following inner product:

If $p=a_0+a_2x^2+a_4x^4$ and $q=b_0+b_2x^2+b_4x^4$,

define $\left\langle p,q\right\rangle = 2a_0b_0+a_2b_2+2a_4b_4$

√ $p_1=1+x^2$

√ $$p_1=1-x^2+x^4-\frac{\left\langle 1-x^2+x^4,1+x^2\right\rangle}{\left\langle 1+x^2,1+x^2\right\rangle}\left(1+x^2\right)$$

$$=1-x^2+x^4-\frac{1}{3}\left(1+x^2\right)$$
$$=\frac{2}{3}-\frac{4}{3}x^2+x^4$$
$$\Rightarrow 2-4x^2+3x^4$$

√ $$p_1=2-x^2-\frac{\left\langle 2-x^2,1+x^2\right\rangle}{\left\langle 1+x^2,1+x^2\right\rangle}\left(1+x^2\right)-\frac{\left\langle 2-x^2,2-4x^2+3x^4\right\rangle}{\left\langle 2-4x^2+3x^4,2-4x^2+3x^4\right\rangle}\left(2-4x^2+3x^4\right)$$

$$=2-x^2-\frac{3}{3}\left(1+x^2\right)-\frac{12}{42}\left(2-4x^2+3x^4\right)$$
$$=2-x^2-\left(1+x^2\right)-\frac{2}{7}\left(2-4x^2+3x^4\right)$$
$$=1-2x^2-\frac{2}{7}\left(2-4x^2+3x^4\right)$$
$$\Rightarrow 7-14x^2-4+8x^2-6x^4$$
$$=3-6x^2-6x^4$$
$$\Rightarrow 1-2x^2-2x^4$$

So, our new basis is $\left\{1+x^2,2-4x^2+3x^4,1-2x^2-2x^4\right\}$