Def: Let *V* be a non-empty set (we often call the elements of this set *vectors*) with an addition operation defined on *V* and scalar multiplication operation defined with scalars from a set *F*. *V* is called a ***vector space over F*** provided each of the following conditions are satisfied:

A1. *Closure under +*: if $\overline{u}, \overline{v} \in V$, then $\overline{u} + \overline{v} \in V$

A2. *Closure under scalar multiplication*: if $\overline{u} \in V$ and $k \in F$, then $k\overline{u} \in V$

A3. *Commutativity for +*: $\overline{u} + \overline{v} = \overline{v} + \overline{u} \quad \forall \overline{u}, \overline{v} \in V$

A4. *Associativity for +*: $\left(\overline{u} + \overline{v}\right) + \overline{w} = \overline{u} + \left(\overline{v} + \overline{w}\right) \quad \forall \overline{u}, \overline{v}, \overline{w} \in V$

A5. *Existence of a zero vector in V*: $\exists\, \overline{0} \in V$ such that $\overline{u} + \overline{0} = \overline{u} \quad \forall \overline{u} \in V$

A6. *Existence of an additive inverse vector in V*:

$$\forall \overline{u} \in V, \quad \exists \left(-\overline{u}\right) \in V \text{ such that } \overline{u} + \left(-\overline{u}\right) = \overline{0}$$

We refer to the vector $-\overline{u}$ as the *additive inverse* of the vector $\overline{u}$

A7. *Unit property*: $\forall \overline{u} \in V, \quad 1 \cdot \overline{u} = \overline{u}$

A8. *Associativity of scalar multiplication*: $\forall \overline{u} \in V$ and $\forall r, s \in F, \quad (rs)\overline{u} = r\left(s\overline{u}\right)$

A9. *Distributive property for scalar multiplication over vector addition*:
$\forall \overline{u}, \overline{v}$ and $\forall r \in F, \quad r\left(\overline{u} + \overline{v}\right) = r\overline{u} + r\overline{v}$

A10. *Distributive property for scalar multiplication over scalar addition*:
$\forall \overline{u}$ and $\forall r, s \in F, \quad (r + s)\overline{u} = r\overline{u} + s\overline{u}$