

Provide both a clear and organized presentation. Show all of your work, completely simplify your answers, and use exact values only. No technology, other than a scientific calculator, can be used to complete this exam.

1. (25 pts) Solve the differential equation: $y^3(x^3 + 8)\frac{dy}{dx} = (x^4 - 8)(y^2 + 2)^{3/2}$

2. (10 pts) Use isoclines to draw a slope field for the differential equation $y' = \frac{y-2}{x}$. Draw and clearly label in your slope field any equilibrium solutions. In addition, draw a couple of additional solutions. What does the existence-uniqueness theorem say about a solution passing through the origin? What does it say about a solution passing through any point other than the origin?

3. (10 pts) Determine the family of orthogonal trajectories to the family of curves described by the following: $y = k \cos x$

4. (15 pts) Solve the differential equation $1 + (y')^2 + y \cdot y'' = 0$

5. (15 pts) Solve the following differential equation:

$$\ln\left(y + \sqrt{y^2 + x^2}\right) - \ln x = \frac{x}{xy' - y} \quad \text{where } x > 0, y > 0$$

6. (15 pts) Solve the differential equation $\frac{dy}{dx} = xy + \frac{x^3}{y}$

7. (10 pts) A farmer's rice field is supplied by two sources of water. For a duration of time, runoff from other nearby irrigated fields supplies this rice field with 100 gallons/hr, but is polluted with 10 g/gallon of assorted pollutants. Another source is constant rain that supplies his field with 50 gallons/hr, but is clean. Water from his field is released at 100 gallons/hr. His field has the capacity to hold 100,000 gallons of water, but is only half full. If his field is initially pollutant-free, then determine a function that gives us the amount of pollutant in this field during the time that the field is filling (i.e., before it reaches its capacity).