

Provide a clear and organized presentation.

1. Let  $n$  be an even natural number and  $A$  an  $n \times n$  matrix. Prove that if the matrix  $B$  is obtained by multiplying every column of the matrix  $A$  whose position is even by the scalar  $k$ , then  $\det(B) = k^{\frac{n}{2}} \det(A)$ .

2. Use *mathematical induction* to prove that: 
$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^n = \begin{bmatrix} f_{n+1} & f_n \\ f_n & f_{n-1} \end{bmatrix} \quad \forall n \in \mathbb{N}$$

where  $f_n$  is the  $n^{\text{th}}$  Fibonacci number (Define  $f_0 = 0$ ).