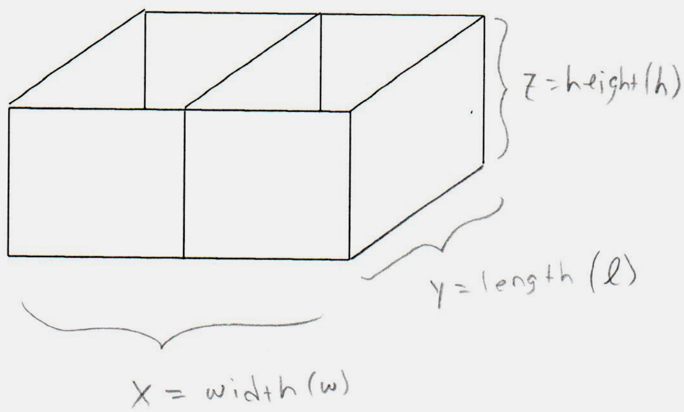
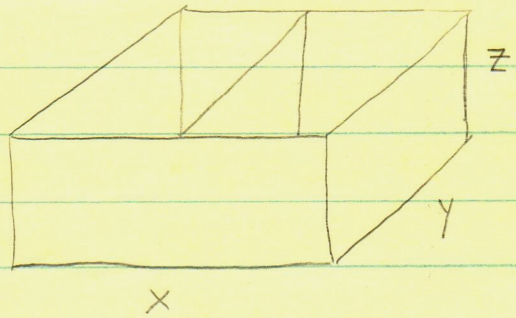
Math 32 Key *Quiz* *October 13, 2017*

Provide a clear and organized presentation. Show all of your work and give exact values only.

A box with an open top has a vertical divider running through its center. The unit cost of the material used for all vertical portions is \$2/ft^2 and the unit cost of the material used for the bottom base is \$3/ft^2. Determine the dimensions that will minimize the cost if the volume is 12 ft^3.

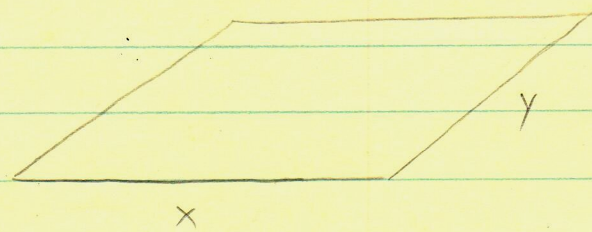




$x, y, z > 0$

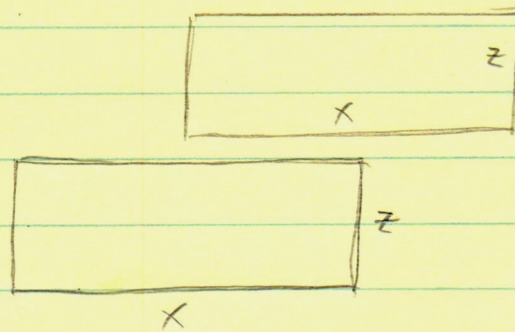
Volume: $xyz = 12$

$$z = \frac{12}{xy}$$

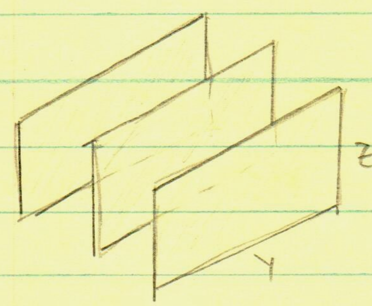


Base
Area : xy

Base
Cost : $3xy$



Front & Back
Area : $2xz$
Cost : $2(2xz) = 4xz$



Sides & Middle divider
Area: $3yz$
Cost: $2(3yz) = 6yz$

Total Cost : $3xy + 4xz + 6yz$

$$C(x,y) = 3xy + 4x\left(\frac{12}{xy}\right) + 6y\left(\frac{12}{xy}\right) = 3xy + \frac{48}{y} + \frac{72}{x}$$

$C_x = 3y - \frac{72}{x^2}$ $\qquad$ $C_y = 3x - \frac{48}{y^2}$

$\downarrow$ $\qquad$ $\downarrow$

$0 = 3y - \frac{72}{x^2}$ $\qquad$ $0 = 3x - \frac{48}{y^2}$ $\qquad$ $0 = 3x - \frac{48}{\left(\frac{24}{x^2}\right)^2}$

$3y = \frac{72}{x^2}$ $\qquad$ $0 = 3x - \frac{48x^4}{(24)^2}$

$y = \frac{24}{x^2}$

$$0 = 3x - \frac{\overset{2}{\cancel{48}}x^4}{\cancel{24}\cdot 24} \longrightarrow 3x - \frac{x^4}{12} = 0 \longrightarrow 36x - x^4 = 0$$

$$x(36 - x^3) = 0$$

$$\cancel{x = 0} \qquad 36 - x^3 = 0$$

Not in domain

$$x^3 = 36$$

$$w \quad x = \sqrt[3]{36}$$

$$y = \frac{24}{x^2} = \frac{24}{(\sqrt[3]{36})^2}$$

$$y = \frac{24}{\sqrt[3]{36^2}} = \frac{24}{\sqrt[3]{2^4 \cdot 3^4}} = \frac{24}{2 \cdot 3\sqrt[3]{6}} = \frac{4}{\sqrt[3]{6}} \text{ OR } \frac{4\sqrt[3]{36}}{6} = \frac{2\sqrt[3]{36}}{3}$$

l

$$C_{xx} = \frac{144}{x^3} > 0 \qquad C_{yy} = \frac{96}{y^3} \qquad C_{xy} = C_{yx} = 3$$

$$C_{xx}C_{yy} - (C_{xy})^2 = \left(\frac{144}{(\sqrt[3]{36})^3}\right)\left(\frac{96}{\left(\frac{4}{\sqrt[3]{6}}\right)^3}\right) - (3)^2$$

$$= \left(\frac{144}{\underset{6}{\cancel{36}}}\right)\left(96 \cdot \frac{6}{64}\right) - 9 = (24)\left(\frac{3}{2}\right) - 9 = 27 > 0$$

Since $C_{xx} > 0$ and $C_{xx}C_{yy} - (C_{xy})^2 > 0$ we have a local / relative minimum when $x = \sqrt[3]{36}$ & $y = \frac{4}{\sqrt[3]{6}}$

Finally to answer the dimension question we need

$$z = \frac{12}{xy} = \frac{12}{(\sqrt[3]{36})\left(\frac{4}{\sqrt[3]{6}}\right)} = \frac{12}{4\sqrt[3]{6}} = \frac{3}{\sqrt[3]{6}} \text{ OR } \frac{3\sqrt[3]{36}}{6} = \frac{\sqrt[3]{36}}{2}$$

h

So to minimize the cost of this box the dimensions should be length $= y = \frac{2\sqrt[3]{36}}{3}$ ft and width $= x = \sqrt[3]{36}$ ft and height $= z = \frac{\sqrt[3]{36}}{2}$ ft

"La grange"

$C(x,y,z) = 3xy + 4xz + 6yz$

$g(x,y,z) = xyz = 12$

$\nabla C = \lambda \nabla g \qquad g = 12$

$\langle C_x, C_y, C_z \rangle = \lambda \langle g_x, g_y, g_z \rangle \qquad g = 12$

$\langle 3y + 4z, 3x + 6z, 4x + 6y \rangle = \lambda \langle yz, xz, xy \rangle$

$3y + 4z = \lambda yz \longrightarrow 3xy + 4xz = \lambda xyz$

$3x + 6z = \lambda xz \longrightarrow 3xy + 6yz = \lambda xyz$

$4x + 6y = \lambda xy \longrightarrow 4xz + 6yz = \lambda xyz$

$xyz = 12 \qquad xyz = 12$

$3xy + 4xz = 12\lambda$

$3xy + 6yz = 12\lambda$

$4xz + 6yz = 12\lambda$

$\rightarrow \cancel{6yz - 4xz = 0 \rightarrow 2z(3y - 2x) = 0}$

$\cancel{z \neq 0} \quad 3y - 2x = 0$

$y = \frac{2}{3}x$

$3xy - 6yz = 0$

$3y(x - 2z) = 0$

$\cancel{y \neq 0} \quad x - 2z = 0$

$x = 2z$

$3xy - 4xz = 0$

$x(3y - 4z) = 0$

$\cancel{x \neq 0} \quad 3y - 4z = 0$

$y = \frac{4}{3}z$

$$x \quad y \quad z = 12$$

$$(2z)\left(\tfrac{4}{3}z\right)(z) = 12$$

$$8z^3 = 36$$

$$z^3 = \frac{9}{2}$$

$$z = \sqrt[3]{\frac{9}{2}} = \frac{\sqrt[3]{9}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{\sqrt[3]{36}}{2} \quad h$$

So $x = 2z = \cancel{2}\left(\frac{\sqrt[3]{36}}{\cancel{2}}\right) = \sqrt[3]{36} \quad w$

$$y = \frac{4}{3}z = \frac{4}{3}\left(\frac{\sqrt[3]{36}}{2}\right) = \frac{2\sqrt[3]{36}}{3} \quad l$$