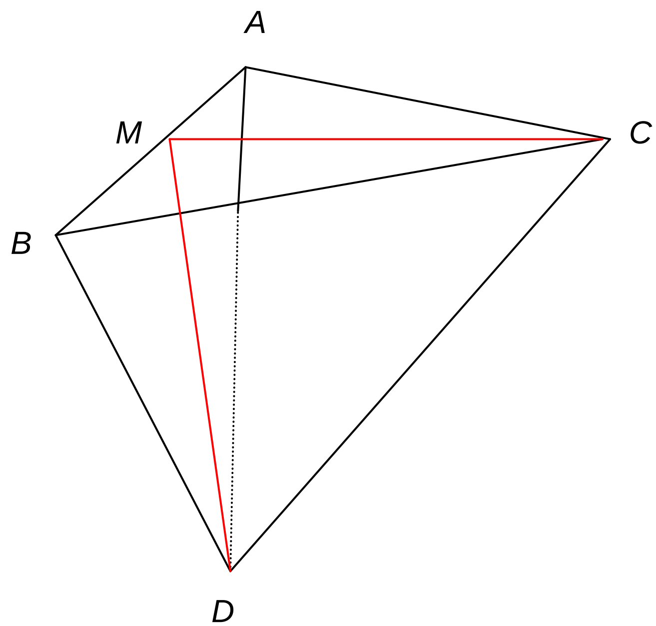
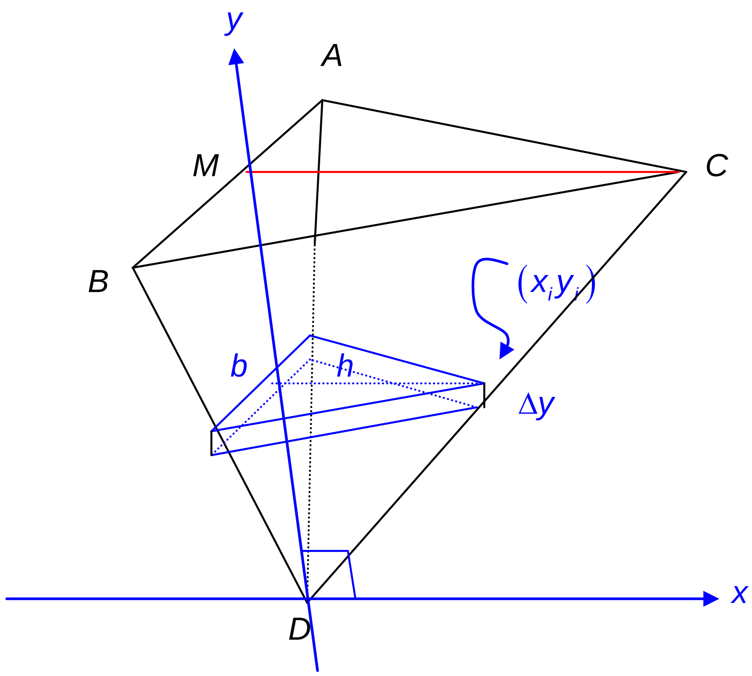
A tank is in the shape of a pyramid as depicted in the following picture. Take into consideration the following qualities that this pyramid possesses: $\overline{CM} \perp \overline{DM}$, $\overline{CM} \perp \overline{AB}$, $\overline{DM} \perp \overline{AB}$, $\overline{AD} \cong \overline{BD}$, and $\overline{AC} \cong \overline{BC} \cong \overline{AB}$. Also note that $DM = 30$ m, $CM = 40$ m, and the triangle ΔABC is parallel to the horizontal.



i) If this tank is full of water, then how much work is required to pump the water to the top, which is open, so that the water flows out?

Reconsider the picture above with the axes provided as I have displayed them (what I have added has been constructed in blue):



- $$v_i = A_{\text{triangle}} \cdot \text{thickness}$$
$$= \frac{1}{2} \cdot \text{base} \cdot \text{height} \cdot \Delta y$$
$$= \frac{1}{2} \cdot b \cdot h \cdot \Delta y \text{ where by similar triangles, } \frac{\sqrt{3}b}{80} = \frac{h}{40}, \text{ or } b = \frac{2}{\sqrt{3}}h$$
$$= \frac{1}{\sqrt{3}} h^2 \Delta y \text{ where, again, by similar triangles, } \frac{h}{40} = \frac{y}{30}, \text{ or } h = \frac{4}{3}y$$
$$= \frac{1}{\sqrt{3}} \left(\frac{4}{3} y_i\right)^2 \Delta y$$
$$= \frac{16}{9\sqrt{3}} y_i^2 \Delta y$$
- $$f_i = \rho \cdot g \cdot v_i$$
$$= 1000(9.8) \frac{16}{9\sqrt{3}} y_i^2 \Delta y$$
$$= \frac{156{,}000}{9\sqrt{3}} y_i^2 \Delta y$$
- $$w_i = f_i d_i$$
$$= \frac{156{,}000}{9\sqrt{3}} y_i^2 \Delta y \cdot (30 - y_i) \qquad *$$
$$= \frac{156{,}000}{9\sqrt{3}} (30 - y_i) y_i^2 \Delta y$$
$$= \frac{156{,}000}{9\sqrt{3}} (30 y_i^2 - y_i^3) \Delta y$$

- $$w=\int_0^{30}\frac{156{,}000}{9\sqrt{3}}\left(30y^2-y^3\right)dy$$

$$=\frac{156{,}000}{9\sqrt{3}}\left(10y^3-\frac{1}{4}y^4\right)\Bigg|_0^{30}$$

$$=\frac{156{,}000}{9\sqrt{3}}\left(270{,}000-\frac{1}{4}\cdot 810{,}000\right)$$

$$=\frac{156{,}000}{9\sqrt{3}}\left(270{,}000-202{,}500\right)$$

$$=\frac{156{,}000}{9\sqrt{3}}\cdot 67{,}500$$

$$=\frac{1{,}170{,}000{,}000}{\sqrt{3}}$$

$$=390{,}000{,}000\sqrt{3}\text{ N-m or Joules}$$

ii) What if the top is closed and there is a spout at the top out from the water flows and the spout is 5 m tall? Assume that the tank is not full, but filled with water whose level is half the height of this tank.

Let's reconsider what to change in the expression *:

$$w_i=\frac{156{,}000}{9\sqrt{3}}y_i^2\Delta y\cdot\left(35-y_i\right)$$ (because the distance to the top of the spout has changed)

and now the integral to compute the total work done becomes:

$$w=\int_0^{15}\frac{156{,}000}{9\sqrt{3}}\left(35y^2-y^3\right)dy$$

$$=\frac{156{,}000}{9\sqrt{3}}\left(\frac{35}{3}y^3-\frac{1}{4}y^4\right)\Bigg|_0^{15}$$

$$=\frac{156{,}000}{9\sqrt{3}}\left(13{,}125-\frac{1}{4}\cdot 1{,}875\right)$$

$$=\frac{156{,}000}{9\sqrt{3}}\cdot\frac{54{,}375-1{,}875}{4}$$

$$=\frac{227{,}500{,}000}{\sqrt{3}}\text{ N-m, or Joules}$$

$$=\frac{682{,}500{,}000\sqrt{3}}{3}\text{ N-m, or Joules}$$