I. Sequence:

1. The sequence $\{a_n\}_{n=1}^{\infty}$ converges if $\lim_{n\to\infty} a_n = L$ for some finite real number L
2. The sequence $\{a_n\}_{n=1}^{\infty}$ diverges if $\lim_{n\to\infty} a_n \neq L$ for some finite real number L

II. Series:

1. Special series:

   a) Consider the *Geometric Series* $\sum_{n=1}^{\infty} ar^{n-1}$:

      i) if $|r| < 1$, this series converges, and its sum is $\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}$,

      ii) if $|r| \geq 1$, this series diverges.

   b) Consider the *P-Series* $\sum_{n=1}^{\infty} \frac{1}{n^p}$:

      i) if $p > 1$, this series converges,

      ii) if $p \leq 1$, this series diverges.

2. TFD (Test for Divergence): The series $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty} a_n \neq 0$

3. Tests for Convergence:

   a) IT (Integral Test): Consider the series $\sum_{n=1}^{\infty} a_n$ and a function *f* for which $f(n) = a_n$ $\forall$ integers $n \geq 1$ where *f* is continuous, positive, and (eventually) decreasing over the interval $[1, \infty)$.

      i) if $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges as well,

      ii) if $\int_1^{\infty} f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges as well.

   b) CT (Comparison Test): Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series with positive terms.

      i) If $a_n \leq b_n$ $\forall n \geq 1$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges

      ii) If $a_n \geq b_n$ $\forall n \geq 1$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges

c) LCT (Limit Comparison Test): Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series with positive terms.

i) If $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ where L is a finite real number, then either both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge or both $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ diverge.

ii) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges as well.

iii) If $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$ and $\sum_{n=1}^{\infty} b_n$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges as well.

d) AST (Alternating Series Test): Given $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ or $\sum_{n=1}^{\infty} (-1)^n a_n$, where both:

i) $a_n \geq a_{n+1} \quad \forall n \geq 1$ and

ii) $\lim_{n\to\infty} a_n = 0$,

then the given series converges.

e) Ratio Test: Given the series $\sum_{n=1}^{\infty} a_n$,

i) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely,

ii) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = L > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges,

iii) If $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$, then any one of the three possibilities might occur: $\sum_{n=1}^{\infty} a_n$ converges absolutely, converges conditionally, or diverges.

d) Root Test: Given the series $\sum_{n=1}^{\infty} a_n$,

i) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely,

ii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$, then $\sum_{n=1}^{\infty} a_n$ diverges,

iii) If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then any one of the three possibilities might occur: $\sum_{n=1}^{\infty} a_n$ converges absolutely, converges conditionally, or diverges.