Provide a clear and organized presentation. Show all of your work and completely simplify your answers. Evaluate each of the following integrals:

1. $\int x^2 \sqrt[3]{x+1}\,dx$ If we let $u = x+1$

$du = dx$

Now we have: $\int x^2 \sqrt[3]{x+1}\,dx = \int (u-1)^2 \sqrt[3]{u}\,du$

$$= \int \left(u^{7/3} - 2u^{4/3} + u^{1/3} \right) du$$

$$= \frac{3}{10}u^{10/3} - \frac{6}{7}u^{7/3} + \frac{3}{4}u^{4/3} + C$$

$$= \frac{3}{140}u^{4/3}\left(14u^2 - 40u + 35\right) + C$$

$$= \frac{3}{140}(x+1)^{4/3}\left(14(x+1)^2 - 40(x+1) + 35\right) + C$$

$$= \frac{3}{140}\left(14x^2 - 12x + 9\right)(x+1)^{4/3} + C$$

2. $\int \frac{e^x \ln\left(e^x + \sqrt{e^{2x}-1}\right)}{\sqrt{e^{2x}-1}}\,dx$ If we let $u = e^x$

$du = e^x dx$

Now we have:

$$\int \frac{e^x \ln\left(e^x + \sqrt{e^{2x}-1}\right)}{\sqrt{e^{2x}-1}}\,dx = \int \frac{\ln\left(u + \sqrt{u^2-1}\right)}{\sqrt{u^2-1}}\,du$$

where we will now let $z = \ln\left(u + \sqrt{u^2-1}\right)$

$$dz = \frac{1}{\sqrt{u^2-1}}\,du$$

$$\int \frac{e^x \ln\left(e^x + \sqrt{e^{2x}-1}\right)}{\sqrt{e^{2x}-1}}\,dx = \int \frac{\ln\left(u + \sqrt{u^2-1}\right)}{\sqrt{u^2-1}}\,du$$

$$= \int z\,dz$$
$$= \frac{1}{2} z^2 + C$$
$$= \frac{1}{2} \ln^2\left(u + \sqrt{u^2 - 1}\right) + C$$
$$= \frac{1}{2} \ln^2\left(e^x + \sqrt{e^{2x} - 1}\right) + C$$

or, we could have used a single substitution, or rather simply state the answer without any substitution because we are well versed with hyperbolic functions.