Provide both a clear and organized presentation. Completely answer each question, give exact values only, and show all of your work. Only a scientific calculator can be used on this exam. Each question is worth 20 points, but there are 6 questions. Choose 5 of these questions to be graded by crossing out one of them. You must cross out which question you would like me to disregard. Otherwise, I will choose the question whose answer is worth the greatest to not grade. Do not use any table of integral reduction formulas.

1. $$\int \frac{2x+4}{6x^3+x^2+2x-1}\,dx = \frac{1}{2}\ln\frac{9x^2-6x+1}{2x^2+x+1} - \frac{1}{\sqrt{7}}\tan^{-1}\frac{4x+1}{\sqrt{7}} + C$$

2. $$\int \frac{x}{x+2\sqrt{x-3}}\,dx = x - 4\sqrt{x-3} + 4\ln\left(x+2\sqrt{x-3}\right) + 2\sqrt{2}\tan^{-1}\frac{\sqrt{x-3}+1}{\sqrt{2}} + C$$

3. $$\int \frac{1}{x+2\sqrt{x^2-3}}\,dx = \frac{1}{2}\ln\left|\frac{\sqrt{x^2-3}}{x}+1\right| - \frac{1}{6}\ln\left|\frac{\sqrt{x^2-3}}{x}-1\right| + \frac{1}{3}\ln\left|\frac{2\sqrt{x^2-3}}{x}+1\right| + C$$

4. $$\int \frac{2\sin x}{\cos x - 4\sin x}\,dx = -\frac{2}{17}\ln\left(t^2+1\right) + \frac{16}{17}\tan^{-1}t + \frac{2}{17}\ln\left|t^2+8t-1\right| + C$$

   where $t = \tan\frac{x}{2}$

5. $$\int \sin^2 x\cos^4 x\,dx = \frac{1}{192}\left(12x - 3\sin 4x + 8\sin^3 2x\right) + C$$

6. Prove that $\int \tan^n x\,dx = \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x\,dx$ if $n \geq 3$ as a natural number

$$\begin{aligned}\int \tan^n x\,dx &= \int \tan^{n-2}x\tan^2 x\,dx \\ &= \int \tan^{n-2}x\left(\sec^2 x - 1\right)dx \\ &= \int \tan^{n-2}x\sec^2 x\,dx - \int \tan^{n-2}x\,dx \\ &= \frac{1}{n-1}\tan^{n-1}x - \int \tan^{n-2}x\,dx\end{aligned}$$