

Show all of your work, be clear and organized, and nowhere on this exam shall we employ the Growth Rate Theorem.

1. (10 pts) Determine the first four terms of the Taylor Series for $f(x) = \frac{1}{x\sqrt{x}}$ centered about $x = 4$.

2. (10 pts) Using what we know about infinite geometric series, determine a power series representation for $f(x) = \ln(2 + x^2)$ both in expanded form and using our summation notation.

3. (10 pts) Determine the radius of convergence for $\sum_{n=1}^{\infty} \frac{(-3)^n (n+3)(2n)!}{n^{2n}} (x-2)^n$

4. (25 pts) Determine whether each of the following series converge. If the series is alternating and converges, determine if that convergence is absolute or conditional.

i)
$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

ii)
$$\sum_{n=1}^{\infty} \frac{1}{3^n - \sqrt{n}}$$

iii)
$$\sum_{n=1}^{\infty} (-1)^n \frac{2^{1/n}}{n!}$$

iv)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 1}{n^3 + 1}$$

v)
$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 1}{3n^2 - 2}$$

5. (10 pts) Use our Analysis of Intervals notation to sketch the graph of the plane curve described by the following parametric equations:

$$x = t^2 - 1$$

$$y = t^3 - 3t$$

7. (15 pts) Consider the equations $r = 2 - 4\cos\theta$ and $r = 3$.
- i) Clearly sketch the polar graphs of these two curves in the same Cartesian coordinate system.
- ii) Determine the area inside the graph of $r = 3$ but outside the graph of $r = 2 - 4\cos\theta$.
- iii) At what values of θ in the interval $[0, 2\pi]$ is $\frac{dy}{dx} = 0$?