

Provide a presentation that is both clear and organized. Show all of your work, completely simplify all answers, and provide exact values only.

1. (12 pts) Let  $f(x) = \frac{\sqrt{12x^3 + 8x^2 - x - 1}}{3x^2 - 7x + 2}$ . Determine the domain of  $f$  in interval notation.

2. (7 pts) Let  $f(x) = \begin{cases} \frac{x^3 - 1}{x - 1} & \text{if } x \neq 1 \\ 5 & \text{if } x = 1 \end{cases}$  Find a function  $g$  that agrees with  $f \forall x \in \mathbb{R}$  except at  $x = 1$  but is continuous  $\forall x \in \mathbb{R}$

3. (16 pts) Evaluate the following limits, if they exist. If they do not, state such. If a limit does not exist as a finite real number but the algebraic expression within the limit increases or decreases without bound, then state such with the appropriate notation.

i) 
$$\lim_{x \rightarrow -\infty} \frac{x-3}{2x + \sqrt{5x^2 + x}}$$

ii) 
$$\lim_{x \rightarrow 0} \frac{(x+3)^5 - 243}{x}$$

4. (16 pts) Evaluate the following limits, if they exist. If they do not, state such. If a limit does not exist as a finite real number but the algebraic expression within the limit increases or decreases without bound, then state such with the appropriate notation.

i) 
$$\lim_{x \rightarrow 0} \frac{\sqrt{2x+3} - \sqrt{3}}{x}$$

ii) 
$$\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{(x-2)^3}$$

5. (14 pts) Use our limit laws to establish that  $f$  is continuous over  $(-\infty, 3]$  if  $f(x) = 2 - \sqrt{3 - x}$  (include one-sided continuity).

6. (10 pts) Let  $f(x) = \begin{cases} \frac{x^2}{2x-c} & \text{if } x \geq 2 \\ 2cx+3 & \text{if } x < 2 \end{cases}$  Determine all values of  $c$  for which  $f$  is continuous over  $(-\infty, \infty)$

7. (15 pts) Determine the equation of the tangent line to the graph of  $y = f(x)$  at  $x = -1$  if  $f(x) = \sqrt{3 - 2x}$ . Put your answer in *slope-intercept* form.

8. (10 pts) Using our precise definition of the limit, prove that  $\lim_{x \rightarrow 2} (5x^2 - x + 2) = 20$