Provide a presentation that is both clear and organized. Show all of your work, completely simplify all answers, and provide exact values only.

1.   (12 pts) Let $f(x) = \dfrac{\sqrt{12x^3 + 8x^2 - x - 1}}{3x^2 - 7x + 2}$. Determine the domain of f in interval notation.

$$\mathrm{dom}f = \left\{-\frac{1}{2}\right\} \cup \left(\frac{1}{3}, 2\right) \cup (2, \infty)$$

2.   (7 pts) Let $f(x) = \begin{cases} \dfrac{x^3 - 1}{x - 1} & \text{if } x \neq 1 \\ 5 & \text{if } x = 1 \end{cases}$ Find a function g that agrees with f $\forall x \in R$ except at $x = 1$ but is continuous $\forall x \in R$

$$g(x) = x^2 + x + 1, \text{ but answers might vary}$$

3.   (16 pts) Evaluate the following limits, if they exist. If they do not, state such. If a limit does not exist as a finite real number but the algebraic expression within the limit increases or decreases without bound, then state such with the appropriate notation.

   i) $\displaystyle\lim_{x \to -\infty} \dfrac{x - 3}{2x + \sqrt{5x^2 + x}} = \dfrac{1}{2 - \sqrt{5}}$

   ii) $\displaystyle\lim_{x \to 0} \dfrac{(x + 3)^5 - 243}{x} = \binom{5}{4} \cdot 3^4 = 405$

4.   (16 pts) Evaluate the following limits, if they exist. If they do not, state such. If a limit does not exist as a finite real number but the algebraic expression within the limit increases or decreases without bound, then state such with the appropriate notation.

   i) $\displaystyle\lim_{x \to 0} \dfrac{\sqrt{2x + 3} - \sqrt{3}}{x} = \dfrac{1}{\sqrt{3}}$

   ii) $\displaystyle\lim_{x \to 2} \dfrac{x^2 - x - 2}{(x - 2)^3} = \infty$

5.   (14 pts) Use our limit laws to establish that f is continuous over $(-\infty, 3]$ if

$f(x) = 2 - \sqrt{3-x}$ (include one-sided continuity).

   i)   First, we will show that f is cts over $(-\infty, 3)$:

$$\text{If } x \in (-\infty, 3), \text{ then } \lim_{x \to a} f(x) = \cdots = f(a)$$

   ii)   Secondly, we will now show left-handed continuity at $x = 3$:

$$\lim_{x \to 3^-} f(x) = \cdots = f(3)$$

6.   (10 pts) Let $f(x) = \begin{cases} \dfrac{x^2}{2x-c} & \text{if } x \geq 2 \\ 2cx + 3 & \text{if } < 2 \end{cases}$ Determine all values of c for which f is

continuous over $(-\infty, \infty)$

$$c = \frac{13 \pm 3\sqrt{33}}{8}$$

7.   (15 pts) Determine the equation of the tangent line to the graph of $y = f(x)$ at

$x = -1$ if $f(x) = \sqrt{3-2x}$. Put your answer in *slope-intercept* form.

$$y = -\frac{1}{\sqrt{5}}x + \frac{4}{\sqrt{5}}$$

8.   (10 pts) Using our precise definition of the limit, prove that $\lim_{x \to 2} (5x^2 - x + 2) = 20$

Hopefully, we had plenty of practice with this definition.