

Provide a clear and organized presentation. Show all of your work, give exact values only, and completely simplify your answers.

1. If  $f(x) = \frac{\sqrt{18x^3 + 3x^2 - 4x - 1}}{x - e}$ , determine the domain of  $f$  in interval notation.

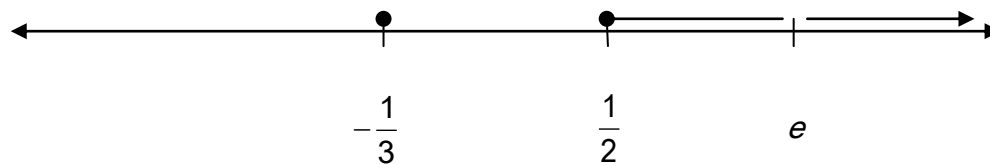
$$\begin{array}{r} \frac{1}{2} \Big| \quad 18 \quad 3 \quad -4 \quad -1 \\ \quad \quad \quad 9 \quad 6 \quad 1 \\ \hline \quad \quad 18 \quad 12 \quad 2 \quad 0 \end{array}$$

So, we now have that:

$$\begin{aligned} f(x) &= \frac{\sqrt{18x^3 + 3x^2 - 4x - 1}}{x - e} \\ &= \frac{\sqrt{\left(x - \frac{1}{2}\right)(18x^2 + 12x + 2)}}{x - e} \\ &= \frac{\sqrt{(2x - 1)(9x^2 + 6x + 1)}}{x - e} \\ &= \frac{\sqrt{(2x - 1)(3x + 1)^2}}{x - e} \end{aligned}$$

Now, we have that  $(2x - 1)(3x + 1)^2 \geq 0$  and  $x \neq e$

On a number line, this gives us:



In interval notation, this gives us:

$$\text{dom } f = \left\{-\frac{1}{3}\right\} \cup \left[\frac{1}{2}, e\right) \cup (e, \infty)$$

2. If  $f(x) = \frac{x}{x^2 - 2}$ , determine  $\frac{f(x+h) - f(x)}{h}$

$$\begin{aligned}
 \frac{f(x+h) - f(x)}{h} &= \frac{\frac{x+h}{(x+h)^2 - 2} - \frac{x}{x^2 - 2}}{h} \\
 &= \frac{(x+h)(x^2 - 2) - x(x^2 + 2hx + h^2 - 2)}{h(x^2 - 2)((x+h)^2 - 2)} \\
 &= \frac{x^3 - 2x + hx^2 - 2h - x^3 - 2hx^2 - h^2x + 2x}{h(x^2 - 2)((x+h)^2 - 2)} \\
 &= \frac{-hx^2 - 2h - h^2x}{h(x^2 - 2)((x+h)^2 - 2)} \\
 &= \frac{-x^2 - 2 - hx}{(x^2 - 2)((x+h)^2 - 2)}
 \end{aligned}$$