Provide a clear and organized presentation. Show all of your work, give exact values only, and completely simplify all answers. Justify all claims of extrema.

1. (10 pts) If $f(x)=3 \sin x$, determine the value of $x$ that satisfies the conclusion of the Mean Value Theorem for $f$ over the interval $\left[0, \frac{\pi}{2}\right]$
2. (10 pts) If $f^{\prime}(x)=90 x^{3}+27 x^{2}-4 x-1$, determine the critical numbers for $f$.
3. (20 pts) Let $f(x)=x+x^{2 / 3}$. Clearly sketch the graph of $y=f(x)$ and label in your graph all intercepts, local extrema, and inflection points.
4. (20 pts) Let $f(x)=\cos x+\cos 2 x$ over $[-\pi, \pi]$. Clearly sketch the graph of $y=f(x)$ over its indicated domain and label in your graph all intercepts and local extrema.
5. (20 pts) A window is constructed in the shape of a rectangle surmounted by an isosceles right triangle as depicted in the following picture. If its perimeter is fixed at 15 ft , then determine the dimensions that will maximize the amount of light that can enter the window.

6. (20 pts) I have a 3 foot wide hallway that meets a 4 foot wide hallway at a right angle. My cat Pythagoras attempts to push on the carpet a stick along one hallway and around the corner to continue down the subsequent hallway. What is the longest length of such a stick that he can get around this corner?

