Provide both a clear and organized presentation. Show all of your work, completely simplify your answers, and use exact values only. No technology, other than a scientific calculator, may be used.

1. (10 pts) If $y = (\cos x)^{\ln x}$, find y'

$$\ln y = \ln(\cos x)^{\ln x}$$
$$\ln y = \ln x \cdot \ln(\cos x)$$
$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}\ln(\cos x) - \tan x \cdot \ln x$$
$$\frac{dy}{dx} = (\cos x)^{\ln x}\left(\frac{1}{x}\ln(\cos x) - \tan x \cdot \ln x\right)$$

2. (10 pts) If $y = \tan^3 \sqrt{x^2 e^{5x}}$, find y'

$$y' = 3\tan^2 \sqrt{x^2 e^{5x}} \cdot \sec^2 \sqrt{x^2 e^{5x}} \cdot \frac{1}{2\sqrt{x^2 e^{5x}}} \cdot \left(2xe^{5x} + 5x^2 e^{5x}\right)$$
$$= \frac{3x(2+5x)e^{5x}\tan^2 \sqrt{x^2 e^{5x}} \cdot \sec^2 \sqrt{x^2 e^{5x}}}{2\sqrt{x^2 e^{5x}}}$$

3. (15 pts) If $x^2 y^3 - 4x + 5y = \sec\frac{y}{x}$, find y'

$$2xy^3 + 3x^2 y^2 \frac{dy}{dx} - 4 + 5\frac{dy}{dx} = \sec\frac{y}{x}\tan\frac{y}{x} \cdot \frac{x\frac{dy}{dx} - y}{x^2}$$
$$2x^3 y^3 + 3x^4 y^2 \frac{dy}{dx} - 4x^2 + 5x^2 \frac{dy}{dx} = \sec\frac{y}{x}\tan\frac{y}{x} \cdot \left(x\frac{dy}{dx} - y\right)$$
$$2x^3 y^3 + 3x^4 y^2 \frac{dy}{dx} - 4x^2 + 5x^2 \frac{dy}{dx} = x\frac{dy}{dx}\sec\frac{y}{x}\tan\frac{y}{x} - y\sec\frac{y}{x}\tan\frac{y}{x}$$
$$\left(3x^4 y^2 + 5x^2 - x\sec\frac{y}{x}\tan\frac{y}{x}\right)\frac{dy}{dx} = 4x^2 - 2x^3 y^3 - y\sec\frac{y}{x}\tan\frac{y}{x}$$
$$\frac{dy}{dx} = \frac{4x^2 - 2x^3 y^3 - y\sec\frac{y}{x}\tan\frac{y}{x}}{3x^4 y^2 + 5x^2 - x\sec\frac{y}{x}\tan\frac{y}{x}}$$

4. (10 pts) If $y = \dfrac{1+x\sqrt{x}}{1-x\sqrt{x}}$, find y'

$$y' = \frac{\left(1-x\sqrt{x}\right)\cdot\frac{3\sqrt{x}}{2} - \left(1+x\sqrt{x}\right)\cdot\left(-\frac{3\sqrt{x}}{2}\right)}{\left(1-x\sqrt{x}\right)^2}$$

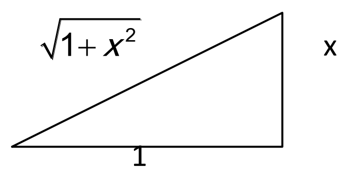
$$= \frac{\left(1-x\sqrt{x}\right)\cdot 3\sqrt{x} - \left(1+x\sqrt{x}\right)\cdot\left(-3\sqrt{x}\right)}{2\left(1-x\sqrt{x}\right)^2}$$

$$= \frac{6\sqrt{x}}{2\left(1-x\sqrt{x}\right)^2}$$

$$= \frac{3\sqrt{x}}{\left(1-x\sqrt{x}\right)^2}$$

5. (10 pts) Derive (i.e., prove that) $\dfrac{d}{dx}\left(\tan^{-1} x\right) = \dfrac{1}{1+x^2}$

Let $y = \tan x$



$\tan y = x$

$\sec^2 y \dfrac{dy}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$

$\dfrac{dy}{dx} = \dfrac{1}{\left(\sqrt{1+x^2}\right)^2}$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$

6. (10 pts) The number of people infected with this year's pesky *Logarithmic Flu Virus* is given by $p(t) = \dfrac{200}{4+e^{-0.2t}}$ where $p(t)$ is measured in hundreds of people and *t* is the number of days beyond the day is broke in this country. What is $p'(10)$ and give a meaningful interpretation of its value.

$$p'(t) = \frac{40e^{-0.2t}}{\left(4+e^{-0.2t}\right)^2} \text{ and } p'(10) \approx 0.31656$$

So, at day 11, we expect approximately 31.7 more people to be infected than the previous day.

7. (15 pts) A tangent line to a curve passes through the point $(8,-2)$. What is the x-coordinate of a point of tangency if the curve is described by the graph of $y = \dfrac{10}{x^2+1}$

$$-11 - \frac{10}{a^2+1} = \frac{-20a}{\left(a^2+1\right)^2}(5-a)$$

$$a^4 + 17a^2 - 80a + 6 = 0$$

With a quick application of one of your favorite theorems, the Rational Root theorem, we clearly see that $a = 3$.

8. (10 pts) If $y = \dfrac{2}{x\sqrt{x}}$, find $y^{(n)}$

$$y = 2x^{-\frac{3}{2}}$$

$$y' = 2\cdot\frac{-3}{2}\cdot x^{-\frac{5}{2}} = -3\cdot x^{-\frac{5}{2}}$$

$$y'' = \frac{3\cdot 5}{2}\cdot x^{-\frac{7}{2}}$$

$$y''' = \frac{-3\cdot 5\cdot 7}{2^2}\cdot x^{-\frac{9}{2}}$$

$$y^{(4)} = \frac{3\cdot 5\cdot 7\cdot 9}{2^3}\cdot x^{-\frac{11}{2}}$$

Consequently, we have that $y^{(n)} = \dfrac{(-1)^n\cdot 3\cdot 5\cdot 7\cdots(2n+1)}{2^{n-1}}\cdot x^{-\frac{2n+3}{2}}$

9. (10 pts) Evaluate the limits:

i) $$\lim_{x\to 0}\frac{\tan^2 5x}{\sin^2 7x}=\lim_{x\to 0}\frac{\sin 5x}{5x}\cdot\frac{\sin 5x}{5x}\cdot\frac{5x}{\cos 5x}\cdot\frac{5x}{\cos 5x}\cdot\frac{7x}{\sin 7x}\cdot\frac{7x}{\sin 7x}=\frac{25}{49}$$

ii) $\lim_{x\to\infty} p(t)$ where $p(t)$ is given in question #6 and give a meaningful interpretation of it.

$$\lim_{t\to\infty} p(t)=\lim_{t\to\infty}\frac{200}{4+e^{-0.2t}}=\frac{200}{4}=50$$

Consequently, 5000 people is the upper limit for the number of people who will have contracted this illness.