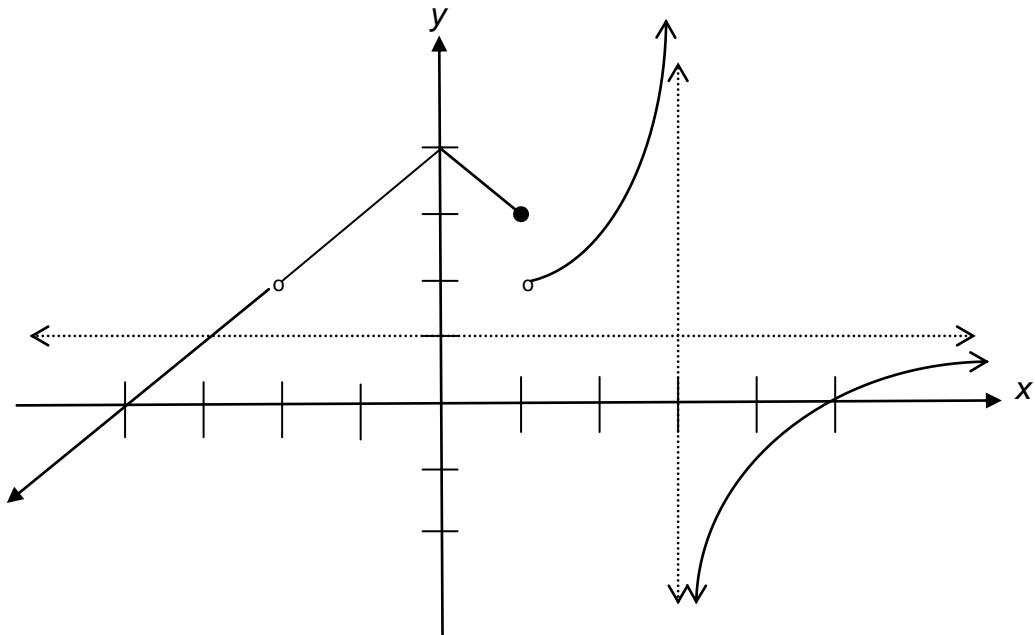


Provide both a clear and organized presentation. Completely answer each question, give exact values only, and show all of your work. Only a scientific calculator can be used on this exam. If any derivative is to be computed, use our definition of the derivative (i.e., the limit of a difference quotient), not any differentiation shortcuts.

1. (16 pts) Consider the graph of  $y = f(x)$  provided below:



Using the graph of  $y = f(x)$  provided above, evaluate each of the following if they exist or are defined. Otherwise, state such.

i)  $\lim_{x \rightarrow -2} f(x) = 2$

ii)  $\lim_{x \rightarrow 1^+} f(x) = 2$

iii)  $\lim_{x \rightarrow 1} f(x)$  d.n.e.

iv)  $\lim_{x \rightarrow 3^+} f(x) = -\infty$

v)  $\lim_{x \rightarrow \infty} f(x) = 1$

vi)  $f(1) = 3$

2. (12 pts) Determine the domain of  $f$  in interval notation if:

$$f(x) = \frac{\sqrt{12x^3 - 8x^2 - x + 1}}{x + \pi}$$

$$12x^3 - 8x^2 - x + 1 \geq 0 \text{ and } x + \pi \neq 0$$

$$(3x+1)(2x-1)^2 \geq 0 \text{ and } x \neq -\pi$$

Consequently,  $\text{dom } f = \left[ -\frac{1}{3}, \infty \right)$

3. (16 pts) If  $f(x) = \frac{x^2}{2-x}$ , find  $f'(x)$

$$\begin{aligned}f'(x) &= \lim_{h \rightarrow 0} \frac{\frac{(x+h)^2}{2-(x+h)} - \frac{x^2}{2-x}}{h} \\&= \lim_{h \rightarrow 0} \frac{(x+h)^2(2-x) - x^2(2-x-h)}{h(2-x)(2-x-h)} \\&= \lim_{h \rightarrow 0} \frac{4x+2h-xh-x^2}{(2-x)(2-x-h)} \\&= \frac{4x-x^2}{(2-x)^2}\end{aligned}$$

4. (8 pts) Let  $f(x) = \begin{cases} \frac{7x}{2x^2 - c} & \text{if } x \geq 2 \\ cx & \text{if } x < 2 \end{cases}$  and determine all values of  $c$  that will allow  $f$  to be continuous over  $(-\infty, \infty)$

$$\lim_{x \rightarrow 2^-} (cx) = \lim_{x \rightarrow 2^+} \frac{7x}{2x^2 - c}$$

$$c^2 - 8c + 7 = 0$$

$$(c-1)(c-7) = 0$$

$$c = 1 \text{ or } c = 7$$

5. (8 pts) Evaluate each of the following limits:

i)  $\lim_{x \rightarrow 2} \frac{|x-2|}{x^3 - 2x^2}$  d.n.e.

ii)  $\lim_{x \rightarrow 2} \frac{x^2 - 5}{(x-2)^2} = -\infty$

6. (32 pts) Evaluate each of the following limits:

$$\begin{aligned}
 \text{i)} \quad & \lim_{x \rightarrow 2} \frac{x^2 - 4}{\sqrt{x} - \sqrt{2}} \\
 &= \lim_{x \rightarrow 2} \frac{(x+2)(\sqrt{x} + \sqrt{2})(\sqrt{x} - \sqrt{2})}{\sqrt{x} - \sqrt{2}} \\
 &= \lim_{x \rightarrow 2} (x+2)(\sqrt{x} + \sqrt{2}) \\
 &= 8\sqrt{2} \\
 \text{ii)} \quad & \lim_{x \rightarrow 2} \frac{x-2}{2x^3 - 3x^2 - 8x + 12} \\
 &= \lim_{x \rightarrow 2} \frac{x-2}{(x-2)(2x^2 + x - 6)} \\
 &= \lim_{x \rightarrow 2} \frac{1}{2x^2 + x - 6} \\
 &= \frac{1}{4} \\
 \text{iii)} \quad & \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 3x + 3} - \sqrt{x^2 + x + 3}}{x} \\
 &= \lim_{x \rightarrow 0} \frac{\sqrt{x^2 + 3x + 3} - \sqrt{x^2 + x + 3}}{x} \cdot \frac{\sqrt{x^2 + 3x + 3} + \sqrt{x^2 + x + 3}}{\sqrt{x^2 + 3x + 3} + \sqrt{x^2 + x + 3}} \\
 &= \lim_{x \rightarrow 0} \frac{2x}{x(\sqrt{x^2 + 3x + 3} + \sqrt{x^2 + x + 3})} \\
 &= \frac{1}{\sqrt{3}} \\
 \text{iv)} \quad & \lim_{x \rightarrow -\infty} \frac{2x + \sqrt{5x^2 + 1}}{3x} \\
 &= \lim_{x \rightarrow -\infty} \frac{2x + \sqrt{5x^2 + 1}}{3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
 &= \lim_{x \rightarrow -\infty} \frac{2x \cdot \frac{1}{x} + \frac{1}{x} \cdot \sqrt{5x^2 + 1}}{3x \cdot \frac{1}{x}} \\
 &= \lim_{x \rightarrow -\infty} \frac{2x \cdot \frac{1}{x} - \sqrt{\frac{1}{x^2}} \cdot \sqrt{5x^2 + 1}}{3x \cdot \frac{1}{x}} \\
 &= \lim_{x \rightarrow -\infty} \frac{2 - \sqrt{5 + \frac{1}{x^2}}}{3} \\
 &= \frac{2 - \sqrt{5}}{3}
 \end{aligned}$$

7. (8 pts) Use our *epsilon-delta* definition of the limit to prove that:

$$\lim_{x \rightarrow 2} (5x^2 - 3x + 1) = 15$$