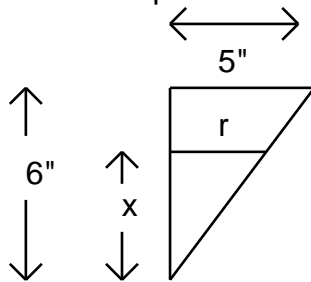


- Find  $\frac{dx}{dt}$ : Relating the quantities  $V$ ,  $r$ , and  $x$  in the cone, we have that:

$$V = \frac{1}{3}\pi r^2 x \quad (1)$$

Note that if we treat  $V$ ,  $r$ , and  $x$  as functions of time  $t$ , then to differentiate the right-hand side w.r.t.  $t$ , we must employ a combination of the product rule and the chain rule. In addition, we will have obtained an equation in

$\frac{dV}{dt}$ ,  $\frac{dr}{dt}$ ,  $\frac{dx}{dt}$ ,  $r$ , and  $x$ . To avoid this, let's rewrite  $V$  solely in terms of  $x$ , and then differentiate implicitly with respect to  $t$ . To accomplish this, consider the following relationship between  $x$  and  $r$  due to our having two triangles that are similar:



By similar triangles, we have that  $\frac{r}{5} = \frac{x}{6}$

$$\text{i.e., } r = \frac{5}{6}x$$

Substituting this expression in  $x$  for  $r$  into equation (1), we have that:

$$V = \frac{1}{3}\pi \left(\frac{5}{6}x\right)^2 x$$

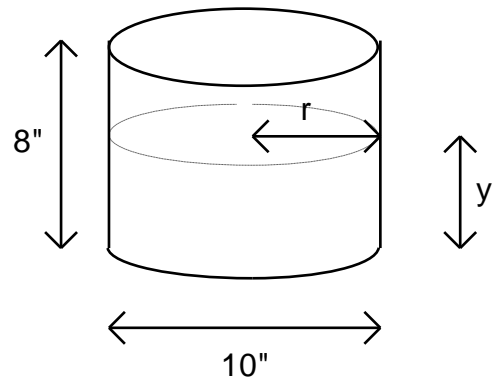
$$V = \frac{25}{108}\pi x^3$$

Now, Treating both  $V$  and  $x$  as functions of time  $t$  and differentiating w.r.t.  $t$ , we see that:

$$\frac{dV}{dt} = \frac{75}{108}\pi x^2 \frac{dx}{dt}$$

Substituting into this equation values known when  $x = 1$  in., we have that:

$$\begin{aligned} -3 &= \frac{75}{108}\pi 1^2 \frac{dx}{dt} \\ \frac{dx}{dt} &= -3 \cdot \frac{108}{75} \cdot \frac{1}{\pi} \\ &= -\frac{108}{25\pi} \text{ in/min.} \\ &\approx -1.38 \text{ in/min.} \end{aligned}$$



- Next, we need to find  $\frac{dy}{dt}$ : Relating the quantities  $V$ ,  $r$ , and  $y$  in the right circular cylinder, note that  $r$  is fixed at 5 inches. So, we have that:

$$V = \pi r^2 y$$

$$V = \pi \cdot 25 \cdot y$$

$$V = 25\pi y$$

Treating both  $V$  and  $y$  as functions in time  $t$  and differentiating with respect to  $t$ , we have:

$$\frac{dV}{dt} = 25\pi \frac{dy}{dt}$$

$$3 = 25\pi \frac{dy}{dt}$$

$$\frac{dy}{dt} = \frac{3}{25\pi} \text{ in/min.}$$

$$\approx 0.04 \text{ in/min.}$$

- Finally, to find the depth of the water  $y$  when  $x = 1$  in., note that  $V_{\text{cylinder}} = 30\pi - V_{\text{cone}}$ . Now, when  $x = 1$  in., we have that:

$$V_{\text{cylinder}} = 30\pi - \frac{1}{3}\pi \cdot \left(\frac{5}{6}\right)^2 \cdot 1$$

$$= 30\pi - \frac{25}{108}\pi$$

$$= \frac{3215}{108}\pi$$

Consequently, we have that:

$$y = \frac{V_{\text{cylinder}}}{\pi r^2}$$

$$= \frac{\frac{3215}{108}\pi}{\pi \cdot 5^2}$$

$$= \frac{643}{540} \text{ in.}$$

$$\approx 1.19 \text{ in.}$$