Math 30 Key **Quiz ?** **October 24, 2017**

Show all of your work and completely simplify your answer.

1. Differentiate (and simplify) $y = \left(1 - \ln(\text{sech}x)\right)\text{sech}x$.

2. Given the curve $f(x) = \text{sech}^{-1}\sqrt{1-x^2}$ $\longrightarrow$ Note: Domain $(-1,1)$ because domain of $\text{sech}^{-1}x$ is $(0,1]$
   a. Find the intervals for which $f'(x) > 0$ and $f'(x) < 0$.
   b. Find all local/relative extrema.
   c. Find the intervals for which $f''(x) > 0$ and $f''(x) < 0$.
   d. Find all inflection points.

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① $y = \left(1 - \ln(\text{sech}x)\right)\text{sech}x$

$$\frac{dy}{dx} = \left(0 - \frac{1}{\cancel{\text{sech}x}}\left(-\text{sech}x\tanh x\right)\right)\cancel{\text{sech}x} + \left(1 - \ln(\text{sech}x)\right)\left(-\text{sech}x\tanh x\right)$$

$$\frac{dy}{dx} = \cancel{\text{sech}x\tanh x} - \cancel{\text{sech}x\tanh x} + \text{sech}x\tanh x\ln(\text{sech}x)$$

$$\boxed{\frac{dy}{dx} = \text{sech}x\tanh x\ln(\text{sech}x)}$$

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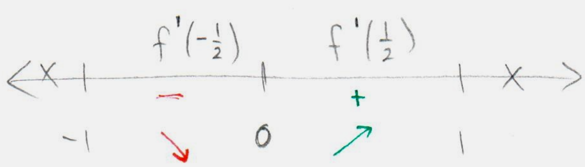
② $f(x) = \text{sech}^{-1}\sqrt{1-x^2}$ Recall: $\frac{d}{dx}\left(\text{sech}^{-1}x\right) = -\frac{1}{x\sqrt{1-x^2}}$

$$f'(x) = -\frac{1}{\cancel{\sqrt{1-x^2}}\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\left(\cancel{\tfrac{1}{2}}\cancel{(1-x^2)^{-1/2}}(-\cancel{2}x)\right) = \frac{x}{(1-x^2)\sqrt{1-\underbrace{|1-x^2|}_{>0 \text{ for } (-1,1)}}} = \frac{x}{|x|(1-x^2)}$$

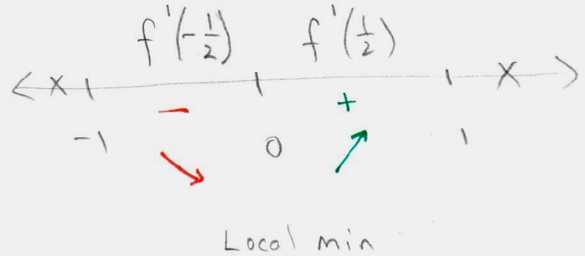
Note: $f'(x) \neq 0$ BUT $f'(x)$ DNE @ $x = 0$.



Thus $f(0) = \text{sech}^{-1}\sqrt{1-(0)^2} = \text{sech}^{-1}(1)$
$f(0) = 0$

a) $f'(x) < 0$ on $(-1,0)$ and $f'(x) > 0$ on $(0,1)$

b) and $(0,0)$ is a local min [No local max]



$f(0) = \operatorname{sech}^{-1}\sqrt{1-(0)^2} = \operatorname{sech}^{-1}(1) = 0$

$f(0) = 0$

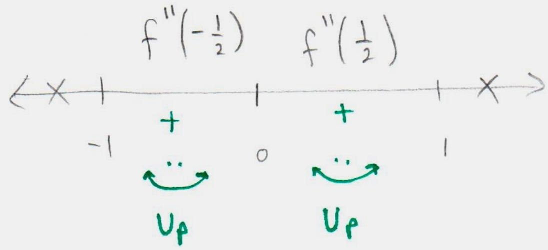
a) $f'(x) < 0$ on $(-1, 0)$ and $f'(x) > 0$ on $(0, 1)$

b) $(0, 0)$ is a local min [No local max]

$$f'(x) = \frac{x}{|x|(1-x^2)} = \begin{cases} \dfrac{1}{1-x^2} & \text{if } x > 0 \\ -\dfrac{1}{1-x^2} & \text{if } x < 0 \end{cases}$$

$$f''(x) = \begin{cases} -(1-x^2)^{-2}(-2x) = \dfrac{2x}{(1-x^2)^2} & \text{if } x > 0 \\ (1-x^2)^{-2}(-2x) = -\dfrac{2x}{(1-x^2)^2} & \text{if } x < 0 \end{cases}$$

DNE @ $x = 0$



c) $f''(x) > 0$ on $(-1, 0) \cup (0, 1)$ [Never $f''(x) < 0$]

d) No inflection points.