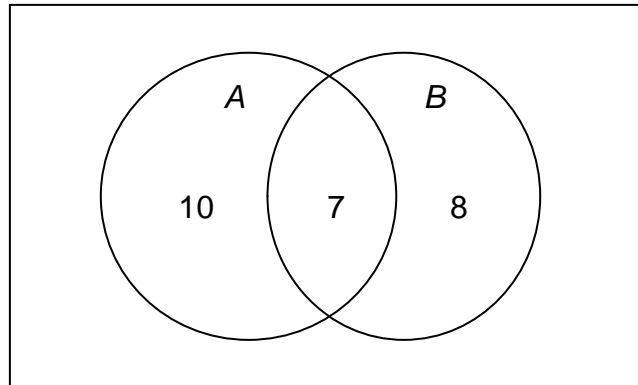


1. Let's investigate the following *Venn diagram* to determine what we will call *the addition property* for sets.



- From the diagram above, it is clear that  $n(A \cup B) \neq n(A) + n(B)$ . Determine a formula for computing  $n(A \cup B)$  in terms of  $n(A)$  and  $n(B)$  where  $A$  and  $B$  are sets that are not disjoint (i.e., where  $A$  and  $B$  are sets that have something in common).
2. Among 80 students, there are 50 taking a mathematics course, 35 taking an English course, and 10 taking both a mathematics course and an English course. How many are taking either a mathematics course or an English course? Count this number two ways: first, with a Venn diagram and second, with the addition property.
3. There are 20,000 students currently attending *Flatland State University*. Through careful surveys, it is determined that 4500 of these students are student athletes. It is also determined that 5200 of these students attending this institution are currently employed. If 8800 students are either student athletes or currently employed, then determine the following by using a Venn diagram representation of the problem situation:
- The number of students who are *both* student athletes and currently employed.
  - The number of students currently enrolled who are not student athletes.
  - The number of student athletes who are not currently employed.
  - The number of students who are both not student athletes and not currently employed.

4. I have an unreasonably large collection of 20 cats roaming my property.
- 8 are talented at performing some trick
  - 8 are adults
  - 9 have a fear of water
  - 3 are both talented and adults
  - 2 are talented and have a fear of water
  - 5 are adults and have a fear of water
  - 2 are talented, adults, and have a fear of water
- i) How many of my cats are talented or are adults, but do not have a fear of water?
- ii) How many of my cats are talented and are adults, but do not have a fear of water?
- iii) How many have no talent, are not adults, and do not have a fear of water?

5. i) Prove pictorially (i.e., with a Venn diagram) the two *DeMorgan's Laws*:

$$\overline{A \cap B} = \bar{A} \cup \bar{B} \text{ and } \overline{A \cup B} = \bar{A} \cap \bar{B}$$

- ii) Illustrate this first version of *DeMorgan's Law* with the following universe and sets:

$$U = \{a, b, c, d, e, f, g, h\}$$

$$A = \{a, b, c\}, B = \{b, c, d, e, f\}$$

6. Consider some universe  $U$  for which the sets  $A$  and  $B$  are subsets. Under what conditions are each of the following true?

i)  $A \cup \emptyset = A$

ii)  $A \cup \emptyset = \emptyset$

iii)  $A \subset (A \cup B)$

iv)  $A \subset (A \cap B)$

v)  $n(A \cup B) = n(A) + n(B)$

7. Let  $A, B \subseteq U$ . Under what conditions is each of the following statements true?

i)  $A \cap B = A$

ii)  $A \cup B = A$

iii)  $\bar{A} \cap B = \emptyset$

iv)  $\overline{A \cap B} = U$

8. Use a *Venn Diagram* model to represent the following universe and sets, then answer the questions that follow. Note that I did not indicate how many students satisfy all three conditions. You might need to consider more than one option/scenario by making some assumptions about how many students satisfy all three conditions.

18 students are surveyed.

10 students are mathematically literate.

4 are highly knowledgeable of world religions.

9 can speak at least three languages fluently.

3 are mathematically literate and are highly knowledgeable of world religions.

5 are mathematically literate and can speak at least three languages fluently.

2 are highly knowledgeable of world religions and can speak at least three languages.

- i) How many of these students are knowledgeable of world religions but are neither mathematically literate nor can speak at least three languages?
- ii) How many of these 18 students are mathematically literate, or highly knowledgeable of world religions, or can speak at least three languages?
- iii) How many of these 18 students are not mathematically literate, nor highly knowledgeable of world religions, nor can speak at least three languages?

9. I have an unreasonably large collection of 15 cats roaming my home.

8 are tailless

6 are afraid of mice

3 have Greek names

5 are both tailless and afraid of mice

3 are tailless and have Greek names

2 are afraid of mice and have Greek names

2 are tailless, afraid of mice, and have Greek names

- i) How many of my cats are tailless, afraid of mice, but do not have Greek names?
- ii) How many are tailless, afraid of mice, or have Greek names?
- iii) How many have tails, fear no mice, and do not have Greek names?