Provide a clear and organized presentation. In class, we proved one half of the *Rational Root Theorem*. Prove the other half.

Let the polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ have a integer coefficients and a rational zero, say *r*. Then $\exists p, q \in Z$ such that $r = \frac{p}{q}$ where, WLOG, we can assume that $\frac{p}{q}$ is completely reduced. Consequently,

$$a_n r^n + a_{n-1} r^{n-1} + \cdots + a_2 r^2 + a_1 r + a_0 = 0$$
$$a_n \left(\frac{p}{q}\right)^n + a_{n-1} \left(\frac{p}{q}\right)^{n-1} + \cdots + a_2 \left(\frac{p}{q}\right)^2 + a_1 \frac{p}{q} + a_0 = 0$$
$$a_n p^n + a_{n-1} p^{n-1} q + \cdots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} + a_0 q^n = 0$$
$$a_{n-1} p^{n-1} q + \cdots + a_2 p^2 q^{n-2} + a_1 p q^{n-1} + a_0 q^n = -a_n p^n$$
$$q\left(a_{n-1} p^{n-1} + \cdots + a_2 p^2 q^{n-3} + a_1 p q^{n-2} + a_0 q^{n-1}\right) = -a_n p^n$$

Note that *q* divides evenly into the left hand side of the equation. Therefore, *q* divides evenly into the right hand side. i.e.,

$$q \mid a_n p^n$$

so, $q \mid a_n$ or $q \mid p^n$

But the latter is a contradiction because $\frac{p}{q}$ is completely reduced.

$\therefore\ q \mid a_n$