Verify each of the following mathematical statements using Mathematical Induction:

1. $\displaystyle\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$ for all $n \in Z^+$

2. $\displaystyle\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ for all $n \in Z^+$

3. $\dfrac{d}{dx}(x^n) = nx^{n-1}$ $\quad \forall n \in N$

4. $1^4 + 2^4 + 3^4 + \cdots + n^4 = \dfrac{n(n+1)(2n+1)(3n^2 + 3n - 1)}{30}$ $\quad \forall n \in N$

5. $6 \cdot 7^n - 2 \cdot 3^n$ is divisible by 4.

6. $\displaystyle\sum_{i=1}^{n} i(i!) = (n+1)! - 1 \quad \forall n \in N$

7. $11^n - 5^n$ is divisible by 3 $\forall n \in N$

8. Let f_n be the n$^{\text{th}}$ Fibonacci number. $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^n = \begin{bmatrix} f_{n-1} & f_n \\ f_n & f_{n+1} \end{bmatrix}$ $\quad \forall n \in N$

9. $\sin x + \sin 3x + \sin 5x + \cdots + \sin(2n-1)x = \dfrac{1 - \cos 2nx}{2\sin x}$ $\quad \forall n \in N$

More Examples:

1. $2^{2n} - 1$ is divisible by 3 $\forall n \in N$

2. $2n + 1 < 2^n$　$\forall n \geq 3$

3. $2^n < (n+1)!$　$\forall n \geq 2$

4. $7^n - 2^n$ is divisible by 5　$\forall n \in N$

5. $3^{2n} - 1$ is divisible by 8　$\forall n \in N$

6. $n^2 < n!$　$\forall n \geq 4$

7. Prove that the sum of the first n terms of a geometric series is given by the following two ways: directly and using *mathematical induction*.

$$S_n = \frac{a(1-r^n)}{1-r} \quad \forall n \in N$$