

Determine if each of the following constitute rings (assume the standard operations of addition, unless otherwise specified):

1. $R = \left\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \right\}$. The key, or work, here is to show closure for both operations.

2. $R = \left\{ \begin{bmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{bmatrix} \mid \delta_{ij} = 1 \text{ if } i = j \wedge \delta_{ij} = 0 \text{ if } i \neq j \right\}$

3. $R = \left\{ \begin{bmatrix} a & b \\ b & a \end{bmatrix} \mid a, b \in R \right\}$

4. $R = \mathbb{R}$ where the addition operation is standard subtraction in the reals.

5. $R = \mathbb{C}$ (i.e., the set of complex numbers). Here, too, concentrate on closure.

6. The set of imaginary numbers.

8. $R = \{0, 1\}$ where the multiplication is normal but addition is defined by the following table:

| | | |
|---|---|---|
| + | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 1 |