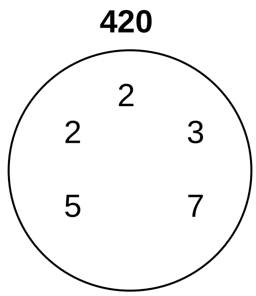
1. If we allow repetitions of prime factors, we can use *Venn diagrams* to display the prime factorization of natural numbers. For example, the prime factorization of 420 is $2 \cdot 2 \cdot 3 \cdot 5 \cdot 7$, and can be displayed as follows:



Use one *Venn diagram* to display the prime factorization of 150 and of 240. What does the union represent? What does the intersection represent?

2. My wife and I inspect computer monitors as they roll off of an assembly line. I inspect every sixteenth monitor, while my wife inspects every thirty sixth. If we both start working at the same time, which monitor will be the first that we both inspect?

3. Archimedes has a collection of 160 pennies and 180 dimes. He desires to place both the pennies and the dimes in stacks of the same number of coins. If each stack contains only one denomination, then what is the largest number of coins that he can place in each stack?

4. What is the greatest common factor of 150 and 480? What is the least common multiple of 150 and 480?

5. Suppose that *p*, *q*, and *r* are prime numbers. Suppose also that $a < b < c$. What is the greatest common factor of $p^a q^b r^c$ and $p^b q^c r^a$? What is the least common multiple of $p^a q^b r^c$ and $p^b q^c r^a$?

6. Suppose that the least common multiple of *p* and *q* is *p*. What can we say about the relationship between *p* and *q*?

7. Suppose that the least common multiple of *p* and *q* is *pq*. What can we say about the relationship between *p* and *q*?

8. Suppose that the greatest common factor of *p* and *q* is *p*. What can we say about the relationship between *p* and *q*?

9. If *p* and *q* are natural numbers and they have a gcf of 5, then determine the lcm of *p* and *q* in terms of *p* and/or *q*.

10. Consider the following (unfinished) work that is used to find the greatest common divisor of 630 and 196:

$630 = 3 \cdot 196 + 42$. i.e., we divide 630 by 196 and note that the remainder is 42. The number 630 is often referred to as the *dividend*, the number 196 as the *divisor*, the number 3 as the *quotient*, and the number 42 as the *remainder*.

Now, repeat this process using the old divisor as the new dividend, and the old remainder as the new divisor:
$196 = 4 \cdot 42 + 28$.

Repeat this process until the remainder is zero. What do you notice about the last nonzero remainder in this process?

11. Use the algorithm from our last exercise, which we will now call the *Euclidean Algorithm*, to determine the gcf of 1155 and 455. Express this gcf as a linear combination of the original two numbers.

12. Use the *Euclidean Algorithm* to determine the gcf of 6942 and 615. Express this gcf as a linear combination of the original two numbers.