Provide both a clear and organized presentation. Show all of your work and give exact values only. A definition of a ring is on the last page of this exam.

1. (15 pts) Prove that if $a = b \bmod 2$ and $a = c \bmod 2$, then $b = c \bmod 2$

2. (25 pts) Determine why each of the following does not form a ring over the indicated operations:

i) $R = \left\{ a \cdot \phi \middle| a \in \mathbb{Q} \wedge \phi \text{ is the golden ratio} \right\}$ with the usual operations of addition and multiplication in $\mathbb{R}$. Recall that $\phi = \dfrac{1+\sqrt{5}}{2}$

ii) $R = \left\{ \dfrac{1}{n} \middle| n \in \mathbb{N} \right\}$ with the usual operations of addition and multiplication in $\mathbb{Q}$.

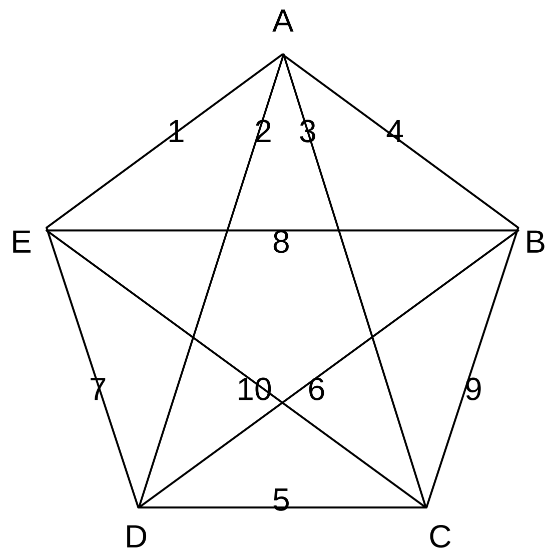
iii) $R = \mathbb{R}$ with the usual operation of addition in $\mathbb{R}$ and multiplication is defined by $a \cdot b = 1 \quad \forall a, b \in \mathbb{R}$.

3. (15 pts) Use an augmented matrix to derive that $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

4. (15 pts) Solve the following system of linear equations by either the inverse matrix method or Cramer's rule:

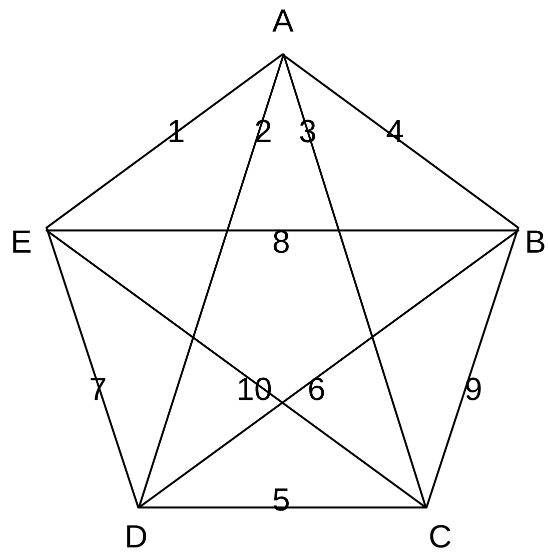
$$2x + y - 3z = -14$$
$$x + 2y + 5z = 24$$
$$3x - y + 2z = 14$$

5. (10 pts) Consider the following graph:



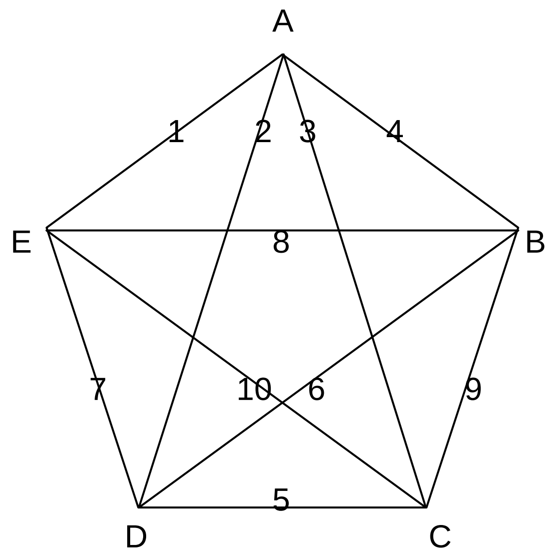
Use the nearest neighbor method to minimize a circuit starting at vertex A. Give its path (i.e., A-E-C-etc.) and its value.

6. (10 pts) Consider the following graph:



Use the cheapest link method to minimize a circuit starting at vertex A. Give its path (i.e., A-E-C-etc.) and its value.

7. (10 pts) Consider the following graph:



Determine the minimal spanning tree for the above graph, draw it below and provide its value.

<u>Def</u>: A <u>ring</u> is a non-empty set R along with two operations $+, \cdot$ satisfying:

$$a+b=b+a \quad \forall a,b \in R$$
$$(a+b)+c=a+(b+c) \quad \forall a,b,c \in R$$
$$\exists e \in R \text{ such that } a+e=e+a=a \quad \forall a \in R$$
$$\forall a \in R, \exists b \in R \text{ such that } a+b=b+a=e$$
$$(a \cdot b)\cdot c=a\cdot(b\cdot c) \quad \forall a,b,c \in R$$
$$a\cdot(b+c)=a\cdot b+a\cdot c \quad \forall a,b,c \in R$$
$$(a+b)\cdot c=a\cdot c+b\cdot c \quad \forall a,b,c \in R$$

Recall that $\mathbb{N}$ = the set of natural numbers
$\mathbb{Q}$ = set of rational numbers
$\mathbb{R}$ = set of real numbers