Math 15	Exam II	March 7, 2018

Provide a clear and organized presentation. Show all of your work, completely simplify your answers, and provide exact values only.

- 1. My cat *Pythagoras* has amassed a large collection of pets. He has 7 lizards, five mice, and 8 snakes. Determine the probability that if he grabs three of these critters with which to play, they will be of the same type.
- 2. Meanwhile, my other cat *Theta* is playing with my coins. He has in his possession one of my unfair coins that has a probability of  $\frac{2}{3}$  for showing *heads*. What is the probability of tensing this gain three times and having a head.

What is the probability of tossing this coin three times and having a *heads* showing at least twice?

3. I have been worried that *Pythagoras* has depression related to the rain. For the past 80 days, I have noted which days have experienced rain and which days during which *Pythagoras* has had a good day. Here are my findings:

	good	bad	
rain	20	10	30
no rain	30	20	50
	50	30	80

Does the rain have an effect on Pythagoras' mood?

- 4. Answer each of the following:
  - i) Let p and q be natural numbers. If gcf(p,q) = 7, then determine lcm(p,q)
  - ii) Let *m* and *n* be natural numbers. Under what conditions is gcf(m,n) = n?
- 5. Let p, q, and r be distinct primes and let a, b, and c be natural numbers for which a < b < c.
  - i) Determine both the  $gcf(p^bq^cr^a, p^cq^ar^b)$  and the  $lcm(p^bq^cr^a, p^cq^ar^b)$
  - ii) If the  $lcm(a,b) = p^8 q^{13}$  and  $a = p^8 q^7$ , then determine a possible representation of *b*.

- 6. Consider the Fibonacci sequence:  $f_1, f_2, f_3, f_4, \dots$  Next, create a new sequence  $d_1, d_2, d_3, d_4, \dots$  for which  $d_n = \frac{f_{n+1}}{f_n}$ . Determine the value of  $\lim_{n \to \infty} d_n$
- 7. In the *Republic of Flatland*, cars are either sedans or coups. In particular, 30% of all cars are sedans and 70% are coups. However, 10% of the sedans are outdated, whereas 20% of all coups are outdated. What is the probability that an arbitrarily chosen car is a sedan if it is known that it is outdated?
- 8. Use the *Euclidean algorithm* to determine gcf(2100,6237) and then rewrite this *gcf* as a linear combination of 2100 and 6237.
- 9. Prove that  $(A \setminus C) \cap (B \setminus C) \subseteq (A \cup B) \setminus C$
- 10. Show that the following is a golden triangle:

