Provide a clear and organized presentation. Show all of your work, completely simplify your answers, and provide exact values only.

1. My cat Pythagoras has amassed a large collection of pets. He has 7 lizards, five mice, and 8 snakes. Determine the probability that if he grabs three of these critters with which to play, they will be of the same type.
2. Meanwhile, my other cat Theta is playing with my coins. He has in his possession one of my unfair coins that has a probability of $\frac{2}{3}$ for showing heads.
What is the probability of tossing this coin three times and having a heads showing at least twice?
3. I have been worried that Pythagoras has depression related to the rain. For the past 80 days, I have noted which days have experienced rain and which days during which Pythagoras has had a good day. Here are my findings:

|  | good | bad |  |
| :--- | :--- | :--- | :--- |
| rain | 20 | 10 | 30 |
| no rain | 30 | 20 | 50 |
|  | 50 | 30 | 80 |

Does the rain have an effect on Pythagoras' mood?
4. Answer each of the following:
i) Let $p$ and $q$ be natural numbers. If $g c f(p, q)=7$, then determine $\operatorname{lcm}(p, q)$
ii) Let $m$ and $n$ be natural numbers. Under what conditions is $\operatorname{gcf}(m, n)=n$ ?
5. Let $p, q$, and $r$ be distinct primes and let $a, b$, and $c$ be natural numbers for which $a<b<c$.
i) Determine both the $g c f\left(p^{b} q^{c} r^{a}, p^{c} q^{a} r^{b}\right)$ and the $\operatorname{lcm}\left(p^{b} q^{c} r^{a}, p^{c} q^{a} r^{b}\right)$
ii) If the $\operatorname{Icm}(a, b)=p^{8} q^{13}$ and $a=p^{8} q^{7}$, then determine a possible representation of $b$.
6. Consider the Fibonacci sequence: $f_{1}, f_{2}, f_{3}, f_{4}, \ldots$ Next, create a new sequence $d_{1}, d_{2}, d_{3}, d_{4}, \ldots$ for which $d_{n}=\frac{f_{n+1}}{f_{n}}$. Determine the value of $\lim _{n \rightarrow \infty} d_{n}$
7. In the Republic of Flatland, cars are either sedans or coups. In particular, $30 \%$ of all cars are sedans and $70 \%$ are coups. However, $10 \%$ of the sedans are outdated, whereas $20 \%$ of all coups are outdated. What is the probability that an arbitrarily chosen car is a sedan if it is known that it is outdated?
8. Use the Euclidean algorithm to determine $\operatorname{gcf}(2100,6237)$ and then rewrite this gcf as a linear combination of 2100 and 6237.
9. Prove that $(A \backslash C) \cap(B \backslash C) \subseteq(A \cup B) \backslash C$
10. Show that the following is a golden triangle:


