

Provide both a clear and organized presentation. Completely answer each question, give exact values only, and show all of your work. Only a scientific calculator can be used on this exam. Unless otherwise specified, each question is worth 12 points. The collective symbols $\exists!$ are banned from this exam.

1. If $A_i = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{i}\right\} \cup \left(-\frac{1}{i}, 0\right]$, determine each of the following:

i) $\bigcup_{i=1}^n A_i$

ii) $\bigcap_{i=1}^n A_i$

2. (4 pts) How many words (not necessarily meaningful ones) can be formed from using all of the letters in the word *totalitarian*?

3. On the distant Island of *Misfit Mathematicians*, people play a version of poker using a strange deck of cards that consists of 6 suits, each containing 8 denominations. In how many ways can one of these islanders draw:

i) four cards of the same suit?

ii) seven cards, four of the same denomination, but that last three distinct from each other and distinct from the denomination of the first four?

4. Let $R(x, y)$ represent the statement: *cat x respects cat y*. Translate each of the following and write your answer in a manner that is as colloquial as possible.

i) $\forall x \exists y R(x, y)$

ii) $\forall y \exists x R(x, y)$

iii) $\exists x \forall y R(x, y)$

iv) $\exists y \forall x R(x, y)$

5. Let $R(x, y)$ represent the statement: *cat x respects cat y* . Translate each of the following using quantifiers:

i) No cat respects itself.

ii) No cat is respected by all others.

iii) All cats respect themselves and exactly one other cat.

6. Prove that $\sqrt{11} \notin \mathbb{Q}$

7. Prove that $3^{2^n} - 2^n$ is divisible by 7 $\forall n \in \mathbb{N}$

8. Prove that $\sum_{i=1}^n 10^i = \frac{1}{9}(10^{n+1} - 10)$ is divisible by 7 $\forall n \in \mathbb{N}$

9. Designate letters to represent the propositions within the following argument, then list both the steps and reasons for those steps. Establish the validity of the argument using our rules of inference.

If my cat *Pythagoras* starts a fire in my house, it will both cause devastation to my household and embarrassment for my family.
My cat *Pythagoras* has not caused embarrassment for my family. If my cat does not start a fire in my house, I will be able to sleep peacefully. Therefore, I sleep peacefully.