Simplify and Solve

Two of the most often used mathematical instructions are simplify and solve. In algebra, we simplify *expressions* and we *solve equations* and *inequalities*.

To simplify an expression, we write it in a less complicated form. To do so, we apply the rules of arithmetic, as well as algebraic concepts such as combining like terms, the distributive property, and the properties of 0 and 1.

To solve an equation or an inequality means to find the numbers that make the equation or inequality true when substituted for its variable. We use the addition, subtraction, multiplication, and division properties of equality or inequality to solve equations and inequalities. Quite often, we must simplify expressions on the left- or right-hand sides of an equation or inequality when solving it.

In Exercises 1-4, use the procedures and the properties that we have studied to simplify the expression in part a and to solve the equation or inequality in part b.

Simplify	ans	Solve	ans
1 a) $-3x + 2 + 5x - 10$	$2x - 8$	b) $-3x + 2 + 5x - 10 = 4$	$x = 6$
2. a) $4(y + 2) - 3(y + 1)$	$y + 5$	b) $4(y + 2) = 3(y + 1)$	$y = -5$
3. a) $\frac{1}{3}a + \frac{1}{3}a$	$\frac{2}{3}a$	b) $\frac{1}{3}a + \frac{1}{3} = \frac{1}{2}$	$a = \frac{1}{2}$
4. a) $-(2x + 10)$	$-2x - 10$	b. $-2x \geq -10$	$x \leq 5$

5. In the student's work on the right, where was the mistake made? Explain what the student did wrong.
The mistake is on the third line. The student made an equation out of the answer (x - 6) by writing "0 =" on the left and then solved that equation.

Simplify $2(x + 3) - x - 12$.

$$2(x + 3) - x - 12 = 2x + 6 - x - 12$$
$$= x - 6$$
$$0 = x - 6$$
$$0 + 6 = x - 6 + 6$$
$$6 = x$$

Adding and Subtracting Real Numbers

To add two real numbers with like signs, add their absolute values and attach their common sign to the sum.

To add two real numbers with unlike signs, subtract their absolute values, the smaller from the larger. To that result, attach the sign of the number with the larger absolute value.

1. Add the numbers
a. -15 + 37
b. 12 + (- 8) + (- 15)
c. -9.9 + (-2.4) -12.3
d. -21 + (- 11) + 32 + (-45)

Properties of the real numbers-the commutative and associative properties of addition:
a + b = b + a (a + b) + c = a + (b + c)

2. Tell what property of addition guarantees that the quantities are equal.
a. -2 + 5 = 5 + (-2) b. (-2 + 5) + 1 = -2 + (5 + 1)

To subtract real numbers, add the opposite:
a - b = a + (-b).

3. Subtract the numbers.
a. 45 - 64 b. - 17 - 32

Solutions of equations can be negative numbers.

4. Solve each equation.
a. $x + 12 = -17$ b. $-1.7 = y - 1.3$

5. SPREADSHEETS Monthly average low temperatures for three cities are listed below. Find SUM(Cl:C3).

	A	B	C	D	E
1	Rockford	15	13	16	26
2	Eagle River	8	-2	-5	18
3	Broadhead	6	4	-8	17

6. GEOGRAPHY The tallest peak on earth is Mt. Everest at 29,028 feet. The greatest ocean depth is the Mariana Trench at -36,205 feet. Find the difference in the two elevations. 65,233 ft

Multiplying and Dividing Real Numbers

When multiplying two real numbers:
1. The product of two real numbers with like signs is positive.
2. The product of two real numbers with unlike signs is negative.

7. Multiply the numbers.
a. $-8 \cdot 7$ b. (-9)(-6) c. 2(-3)(-2)
d. (-3)(4)(2) e. (-3)(-4)(-2)

Properties of the real numbers-the commutative and associative properties of multiplication:

$$ab = ba$$
$$(ab)c = a(bc)$$

8. Tell what property of multiplication guarantees that the quantities are equal.
a. $(2 \cdot 3)5 = 2(3 \cdot 5)$ b. (-5)(-6) = (-6)(-5)

When dividing two real numbers:
1. The quotient of two real numbers with like signs is positive.
2. The quotient of two real numbers with unlike signs is undefined negative.

9. Do each division.
a. $\frac{88}{44}$ b. $\frac{-100}{25}$ c. $\frac{-81}{-27}$
d. $\frac{0}{37}$

Order of Operations and Evaluating Algebraic Expressions

An exponent is used to indicate repeated multiplication.

10. Find each power.
a. 2^5 b. $(-2)^5$ c. $(-3)^4$

Order of operations:
Work from the innermost pair of grouping symbols to the outermost pair in the following order:

1. Evaluate all exponential expressions.
2. Do all multiplications and divisions, working from left to right.
3. Do all additions and subtractions, working from left to right. If the expression does not contain grouping symbols, begin with step 1. In a fraction, simplify the numerator and denominator separately. Then simplify the fraction, if possible.

11. Evaluate each expression.

a. $4^3 + 2(-6 - 2 \cdot 2)$ b. $-5[-3 - 2(5 - 7^2)] - 5$

c. $(-3)^3\left(\frac{-8}{2}\right)+5$

12. Evaluate $3(x - y) - 5(x + y)$ when $x = 2$ and $y = -5$

Simplifying Algebraic Expressions

To simplify an algebraic expression" means to write it in less complicated form.

13. Simplify each expression

a. $-4(7w)$ b. $-3r(-5r)$

The distributive property:
$a(b + c) = ab + ac$
$a(b - c) = ab - ac$

14. Write each expression without parentheses.

a. $5(x + 3)$ b. $-2(2x + 3 - y)$

In a term, the numerical factor is called the coefficient.

15. Identify the coefficient of each term

a. $2x - 5$ b. $16x^2 - 5x + 25$

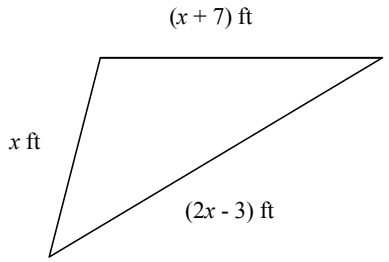
Like terms are terms with exactly the same variables and exponents.

16. Simplify each expression by combining like terms.

a. $8p + 5p - 4p$ b. $-5m + 2n - 2m - 2n$

c. $5(p - 2q) - 2(3p + 4q)$

17. Write an algebraic expression in simplified form for the perimeter of the triangle shown below.



Solving Equations

To solve an equation means to find all the values of the variable which, when substituted for the variable, make a true statement.

18. Solve each equation.

a. $5x + 4 = 14$

b. $-12y + 8 = 20$

c. $5(2x - 4) - 5x = 0$

d. $-2(x - 5) = 5(-3x + 4) + 3$

An equation that is not true for any values of its variable is called an impossible equation.

19. SOUND SYSTEM A 45-foot-long speaker wire is to be cut into three pieces. One piece is to be 15 feet long. Of the remaining pieces, one must be 2 feet less than 3 times the length of the other. Find the length of the shorter piece of wire.

Problem Solving

To solve problems, use the five-step problem-solving strategy.

1. Analyze the problem
2. Form an equation
3. Solve the equation
4. State the conclusion
5. Check the result

20. UTILITY BILLS The electric company charges $17.50 per month, plus 18 cents for every kilowatt hour of energy used. One resident's bill was $43.96. How many kilowatt hours were used that month?

21. INVESTMENT INCOME A woman has $27,000. Part is invested for one year in a certificate of deposit paying 7% interest, and the remaining amount in a cash management fund paying 9%. After 1 year, the total interest on the two investments is $2,110. How much did she invest at each rate?

Distance = rate · time
$d = rt$

22. WALKING AND BICYCLING A bicycle path is 5 miles long. A man walks from one end at the rate of 3 mph. At the same time, a friend bicycles from the other end, traveling at 12 mph. In how many minutes will they meet?

Inequalities

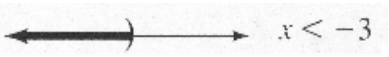
An inequality is a mathematical expression that contains a >, <, ≥, ≤, =, or ≠ symbol

A solution of an inequality is any number that makes the inequality true.

For instance

1) $3x + 2 < 5$

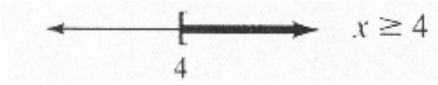
2) $-5x - 8 > 7$



A parenthesis indicates that a number is not on the graph. A bracket indicates hat a number is included in the graph.

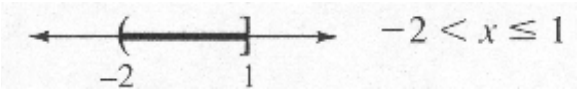
For instance

1) $5x - 3 \geq 2x + 9$



2) $7x + 1 \leq 8x - 5$

3) $0 \leq 2 - 2x < 6$



23. Solve each inequality and graph the solution.

a. $5(3 - x) \leq 3(x - 3)$ b. $8 < x + 2 < 13$ c. $0 \leq 2 - 2x < 6$

Interval notation can be used to describe a set of real numbers.

24. Graph the interval represented by [- 13, x).

Quiz

1 . Add (-6) + 8 + (-4)

2. Subtract 1.4 - (-0.8)

3. Multiply (-2)(-3)(-5)

4. Evaluate $-7[(-5)^2 - 2(3 - 5)^2]$

5. Evaluate $\dfrac{3(20 - 4^2)}{-2(6 - 2^2)}$

In Problems 6 & 7, let x - 2, y = 3, and z = 4. Evaluate each expression.

6. $xy + z$

7. $x(y + z)$

8. What is the numerical coefficient of the term 6x?

9. How many terms are in the expression $3x^2 + 5x - 7$?

In Problems 10-12, simplify each expression.

10. 5(-4x)

11. $-6x(-7y)$

12. $3(x + 2) - 3(4 - x)$

In Problems 13-14, solve each equation.

13. $2(x - 7) - 15$

14. $\dfrac{3(x - 6)}{2} = 6x$

In Problems 15 and , 16 solve each equation for the variable indicated.

15. $d = rt$; for t

16. $A = P + Prt$; for r

17. MIXTURE PROBLEM How many liters of a 2% brine solution must be added to 30 liters of a 10% brine solution to dilute it to an 8% solution?

In Problems 18 and 19, solve each inequality and graph its solution.

18. $-8x - 20 \leq 4$

19. $-4 \leq 2(x + 1) < 10$

Teamwork

SOLVING EQUATIONS Make a presentation to the class explaining how we "undo" operations to isolate the variable when solving an equation. As a visual aid, bring in a box tied shut with string that contains a toy wrapped in tissue paper. Compare the three-step process a person would use to get to the toy inside the box to the three-step process we could use to solve the equation $\frac{2x}{3} - 4 = 2$

MIXTURES Get several cans of the same brand of orange juice concentrate and make up four pitchers of concentrate and water mixtures that are 10%, 30%, 50%, and 70% orange juice concentrate. For example, a 30% solution would consist of three small paper cups of concentrate and seven small paper cups of water. Pour small amounts of each mixture into clean cups. Have students not in your group taste each solution and see whether they can put the mixtures in order from least concentrated to most concentrated.

EXPONENTS Use a scientific calculator and the exponential key 7 (on some calculators, it is labeled to decide whether each statement is true or false.

1. $7^5 = 5^7$

2. $2^3 + 7^3 = (2 + 7)^3$

3. $(-4)^4 = -4^4$

4. $\frac{10^3}{5^3} = 2^3$

5. $8^4 \cdot 9^4 = (8 \cdot 9)^4$

6. $2^3 \cdot 3^3 = 6^3$

7. $\frac{3^{10}}{3^2} 3^5$

INEQUALITIES In most states, a person must be at least 16 years of age to be eligible to apply for a driver's license. We can mathematically describe this situation with the inequality $a \geq 16$, where a represents a person's age in years. Think of other situations that can be described using an inequality. Try to come up with some examples that require compound inequalities.

Five students, All-star, Brainy, Cinch., Dummy, and Egghead. find that each has books that one or more of the others can use this term. Each lends out four books and borrows four books. No two students lend their books in exactly the same numerical combination. All-star borrowed all four of Brainy's books. Cinch lent three books to Egghead. From whom did Dummy borrow his four and how many did he borrow from each?

Two people are standing together. They start walking with their right feet together. They walk together, but for every three steps, one person takes the other takes four. When will they be in step again?

Three sailors went to a hotel and paid $30 for a room. After they had gone up to the room, the desk clerk discovered that they were in a $25 room and sent the bellhop up to refund the $5 overpayment. The bellhop realized that $5 would be hard to divide among the three sailors, and so (to keep good feelings among the sailors) pocketed $2 and returned only $3 to the sailors.

Sailors paid $30 - $ 10 per sailor;
Sailors were refunded $3 - $1 per sailor;
Bellhop kept $2.

Thus, the sailors paid $9 each, or a total of $27. The bellhop kept $2, which makes a total of $29. However, the sailors paid $30 originally. Where did the missing dollar go?

The 3-4-5 right triangle described in this section is a special case, because all the sides are integers. However, there are more all-integer right triangles. These sets
of sides are called Pythagorean Triples, and one sequence of these triples begins as follows.

$3^2 + 4^2 = 5^2$	$9 + 16 = 25$
$5^2 + 12^2 = 13^2$	$25 + 144 = 169$
$7^2 + 24^2 = 25^2$	$49 + 576 = 625$
$9^2 + 40^2 = ?$	$81 + 1600 = ?$
$11^2 + ? = ?$	?
$13^2 + ? = ?$	
$? + ? = ?$	

Can you see a pattern? Show the next few parts of these Pythagorean Triples.

A rope is laid on the ground around the equator of the earth. A cut is made in the rope and an additional yard of rope is added to this rope. The lengthened rope now forms a larger circle hovering above the earth's surface.

i) Intuitively, will this lengthened piece of rope be significantly above the ground?

ii) Recall the formula for computing the circumference of a circle. Give an expression for C_E, the circumference of the earth, in terms of r_E, the radius of the earth.

iii) Let $x =$ the distance from the ground for which the new rope resides. Give <u>two</u> algebraic descriptions for C_L, the lengthened piece of rope, one involving r_E and x, the other involving r_E and the one-yard length.

iv) Use both of the above descriptions of C_L to algebraically answer the question: what is the height above the equator's surface at which this lengthened piece of rope resides?

Without using a calculator, determine which of the following, if any, is the largest quantity:

$$(-2)^{79}$$
$$-1\cdot 2^{79}$$
$$(-1)^{79}\cdot 2$$
$$-1\cdot(-2)^{79}$$
$$\left|(-1)^{79}\right|\cdot 2$$

Key Real Numbers

1. a. 22
2. a. Commutative property of addition; b. associative property of addition
3. a. –19; b. –49
4. a. –29
5. 15
6. 65,233 ft
7. a. –56; b. 54; c. 12; d. –24
8. a. associative property of multiplication; b. commutative property of multiplication
9. a. 2; b. –4; c. 3; d. 0
10. a. 32; b. –32; c. 81; d. –125
11. a. 44; b. –430; c. $-\frac{14}{19}$; d. 113
12. a. 36
13. a. $-28w$; b. $15r^2$
14. a. $5x + 15$; b. $-4x - 6 + 2y$
15. a. 2, -5; b. 16, -5, 25
16. a. $9p$; *b*. $-7m$; c. $-2a - 10b$; d. $-p - 18q$
17. $(4x + 4)$ ft
18. a. 2; b. –1; c. 4; d. 1
19. 8 ft
20. 147
21. $16,000 at 7%, $11,000 at 9%
22. 20