VARIABLES

One of the major objectives of this course is for you to become comfortable working, with variables. Four different approaches will introduce the topic of variables.

Stating mathematical properties
Variables have been used to state properties of mathematics in a concise, "shorthand" notation.

1. Complete the statement of the subtraction property of equality: Let a, b, and c be any three numbers. If a = b, then a - c must equal b - c

2. Complete the statement of the multiplication property of equality: Let a, b, and c be any three numbers ($c \neq 0$). If a = b, then ca must equal cb

Writing algebraic expressions
Variables and numbers (called constants) have been combined with the operations of addition, subtraction, multiplication, and division to create algebraic expressions.

3. One year, a cruise company did x million dollars worth business. After a television, celebrity was signed as a spokeswoman for the company, its business increased by $4 million the next year. Express the amount of business the cruise company had in the year the celebrity was the spokeswoman.

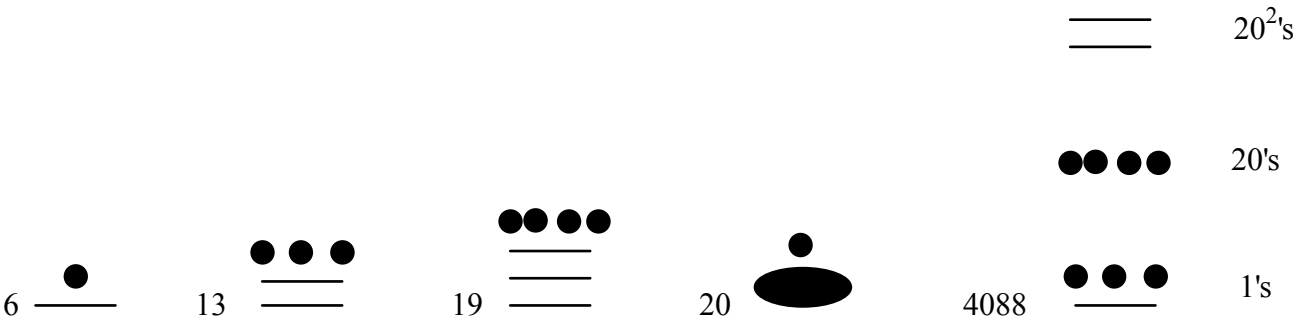
x + 4 = amount of business ($ millions) the year with the celebrity

ZERO AND MAYAN MATHEMATICS

The invention of zero occurred independently in two different parts of the world. The earliest known use of zero in India is in a Hindu inscription Of A.D. 876. Through Arab scholars, knowledge of zero was transmitted to Europe.

Between 300 B.C. and A.D. 300, the Mayan Indians of Central America had performed many amazing astronomical calculations, such as determining the orbit of Venus with an error of I day in 6,012 years. These results have led many archaeologists to believe that the Mayans were the first people to invent zero. The evidence for this is strong, but circumstantial, as the sixteenth century Spanish conquerors destroyed nearly all of the Mayan written records, believing them to be "the work of the devil." The symbol the Mayans used for zero was a shell,

Mayan merchants used a base-20 number system. They wrote their numbers vertically, so that as one moved up the column, the numbers increased by a factor of 20. Bar and-dot numerals were used; the dot (•) represented 1, and the bar (-) represented 5. In our base-10 system we write the numbers 1 to 9 and then use our zero symbol to write 10. In the based-20 system the Mayans wrote the numbers I to 19 and then used their zero symbol to write 20:



The last number written above is expressed in our Hindu-Arabic system as $10 - 20^2 + 4 \cdot 20 + 8 \cdot 1 = 4088$. The simplicity of Mayan addition is illustrated with these numbers. The sum of 6 and 13 is found by combining all the dots and bars of 6 and 13, giving the numeral for 19. The accompanying figure from a Mayan manuscript illustrates the mixture of religious text and arithmetic.

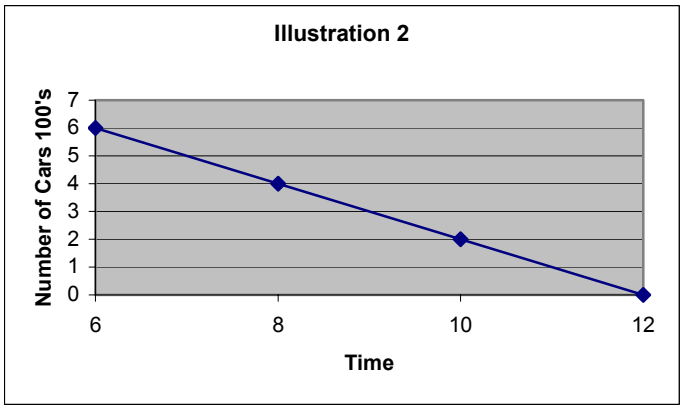
CONCEPTS

Tables, bar graphs, and line graphs are used to describe numerical relationships.

1. On the table to the right lists the worldwide production of wide-screen TVs. Use the data to construct a bar graph. Describe the trend in the production in words.

YEAR	PRODUCTION (MILLIONS OF UNITS)
'95	3
'96	5
'97	7
'98	11

2. Consider the line graph in Illustration 2 that shows the number of cars parked in a mail parking structure from 6 P.M. to 12 midnight on a Saturday.



a. What units are used to scale the horizontal and vertical axes?

b. How many cars were in the parking structure at 11 P.M.?

c. At what time did the parking structure have 500 cars in it?

The result of an addition is called the sum; of a subtraction, the *difference;* of a multiplication, the *product;* and of a division, the *quotient.*

3. Express each statement in words.
a. 15 - 3 = 12
b. 15 + 3 = 18
c. 15 - 3 = 5
d. 15 - 3 = 45

In algebra, we use many new symbols and notations.

4. a. Write the multiplication 4×9 in two ways: first with a raised dot then using parentheses.
b. Write the division $9 \div 3$ without using the symbols ÷ or $\sqrt{\ }$

5. Write each multiplication without a multiplication symbol.

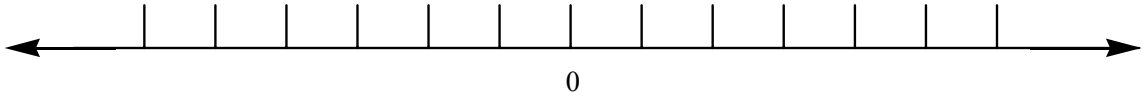
a. $8 \cdot b$ b. $x \cdot y$ c. $2 \cdot l \cdot w$ d. $P \cdot r \cdot t$

REAL NUMBERS

The natural numbers: {1, 2, 3, 4, 5, 6, . . .}
The whole numbers: {0, 1, 2, 3, 4, 5, 6,…}
The integers: {1. . . , -3, -2, - 1, 0, 1, 2, 3, . . .}

6. According to the group of numbers on the left, which number is a whole number but not a natural number?

7 . Graph each member of the set { - 3, 5, 0, -1} on the number line.



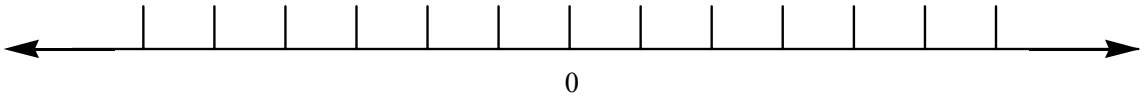
Two inequality symbols are > "is greater than" < "is less than"

8. Use one of the symbols > or < to make each statement true.

a. 0 5 b. 6 4 c. -12 -13 d. -3 2

Rational numbers are either terminating or repeating decimals.

9. Graph each member of the set $\{-\pi, 0.333\ldots.3.75, \frac{-17}{4}, \frac{7}{8}\}$



A real number is any number that is either a rational or an irrational number.

10. Tell whether each statement is true or false.
   a. All integers are whole numbers. false
   b. π is an irrational number. true
   c. The real numbers are the set of all decimals. true
   d. A real number is either rational or irrational. true

The natural numbers are a subset of the whole numbers. The whole numbers are a subset of the integers. The integers are a subset of the rational numbers.

11. Tell which numbers in the given set are natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

$$\{\frac{-4}{5}, 99.99, 0, \sqrt{2}, -12, 4\frac{1}{2}, 0.666\ldots 8\}$$

The absolute value of a number is the distance on the number line between the number and 0.

12. Insert one of the symbols >, <, or = in the blank to make each statement true.
    a. $|-6|$ $\quad$ $|5|$
    b. $|-9|$ $\quad$ $-|-10|$
    c. $|-39|$ $\quad$ 39
    d. $|-.25|$ $\quad$ 0.333

EXPONENTS AND ORDER OF OPERATIONS

An exponent is used to represent repeated multiplication. In the exponential expression a^n, a is the base, and n is the exponent.

13. Write each expression using exponents….Try Your Best

a. $8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$ $\quad$ b. $(2)(2)(2)$

c. $5 \cdot 5 \cdot 5 \cdot 9 \cdot 9$ $\quad$ d. $a \cdot a \cdot a \cdot a \cdot a$

e. $9 \cdot \pi \cdot r \cdot r \cdot r$ $\quad$ f. $x \cdot x \cdot x \cdot y \cdot y \cdot y \cdot y$

g. one hundred squared $\quad$ h. the sixth power of one

Order of operations

If an expression contains grouping symbols, do all calculations working from the innermost pair to the outermost pair, in the following order:

1. Evaluate all exponential expressions.
2. Do all multiplications and divisions, working from left to right.
3. Do all additions and subtractions, working from left to right. If the expression does not contain grouping symbols, begin with step 1.

14. How many operations does the expression $5 \cdot 4 - 3^2 + 1$ contain, and in what order should they be performed?

In a fraction, simplify the numerator and the denominator separately. Then simplify the fraction, whenever possible.

15. Evaluate each expression.
    a. $24 - 3 \cdot 6$
    b. $6 \cdot 5 - 18 \div 9$
    c. $800 - 3 \cdot 4^4$
    d. $\dfrac{8^2 - 10}{2(3)(4) - 2 \cdot 3^2}$

ALGEBRAIC EXPRESSIONS

In order to describe numerical relationships, we need to translate the words of a problem into mathematical symbols.

16. Write each phrase as an algebraic expression
    a. 25 more than the height h
    b. 15 less than the cutoff scores *s*
    c. The product of 6 and x

When we replace the variable, or variables, in an algebraic expression with specific numbers and then apply the rules for the order of operations, we are evaluating the algebraic expression.

17.Complete the following table

x	$20x - x^3$
0	
1	
4	

18. Evaluate each algebraic expression for the given value(s) of the variable(s).
   a. $6x$ for $x = 6$
   b. $7x^2 - \frac{x}{2}$ for $x = 4$
   c. b^2-$4ac$ for $b = 10$, $a = 3$
   d. $2(24\text{-}2c)^3$ for $c = 9$

An equation is a statement indicating that two expressions are equal. Any number that makes an equation true when substituted for its variable is said to satisfy the equation. Such numbers are called solutions or roots.

19. a. $x - 34 = 50$; $x = 80$
   b. $5y + 2 = 12$; $y = 3$
   c. $5b - 2 = 3b + 3$; $b = 3$
   d. $a^2 - a - 1 = 0$; $a = 2$

20. Solve each equation. Check all solutions.

   a. $x - 9 = 12$
   b. $y + 15 = 32$
   c. $4 = v - 1$
   d. $100 = 7 + x$

The percent formula:

We can translate a percent problem from words into an equation. A variable is used to stand for the unknown number; is can be translated to an = sign; and of means multiply

Amount = percent · base.

21. FAMILY BUDGET It is recommended that a family pay no more than 30% of its monthly income (after taxes) on housing. If a family has an after-tax income of $1,890 per month and pays $625 in housing costs each month, are they within the recommended range?

PROBLEM SOLVING

An equation is a mathematical statement that two quantities are equal.

To solve a problem, follow these steps:
1. Analyze the problem.
2. Form an equation.
3. Solve the equation.
4. State the conclusion
5. Check the result.

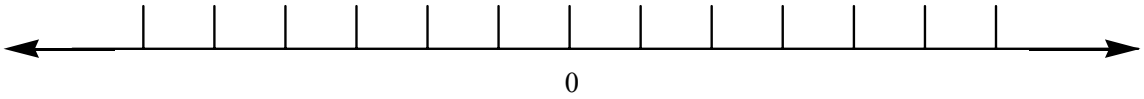
22. HISTORIC TOUR A driving- tour of three historic cities is an 858-mile round trip. Beginning in Boston, the drive to Philadelphia is 296 miles. From Philadelphia to Washington, DC is another 133 miles. How long would the return trip to Boston be?

QUIZ

1. Use the formula $f = \frac{a}{5}$ to complete the table (a = area, f = fire stations).

Area in square miles	Number of fire stations
15	
100	
	70

2. Graph each member of the set $\{1\frac{1}{4},\ \sqrt{2}\ ,\ -3.75,\ \frac{7}{2},\ 0.5\}$



3. Insert one of the symbols >, <, or = in the blank to make each statement true.
   a. $|-2|$ ____ $|-3|$
   b. $-|-7|$ ____ $|8|$

4. Write each expression using exponents.
a. $9 \cdot 9 \cdot 9 \cdot 9 \cdot 9$
b. $3 \cdot x \cdot x \cdot x \cdot y$

5. Evaluate the expression 5^6

6. Evaluate the expression $\dfrac{4^3 - 2 \cdot 13}{4\frac{25}{5} - 1^4}$

7. Evaluate the algebraic expression for the given value of the variables.

$2Lw + w^2$ for $l = 6$ and $w = 8$

8. Solve $x + 11 = 24$

9. Solve $\frac{c}{10} = 55$

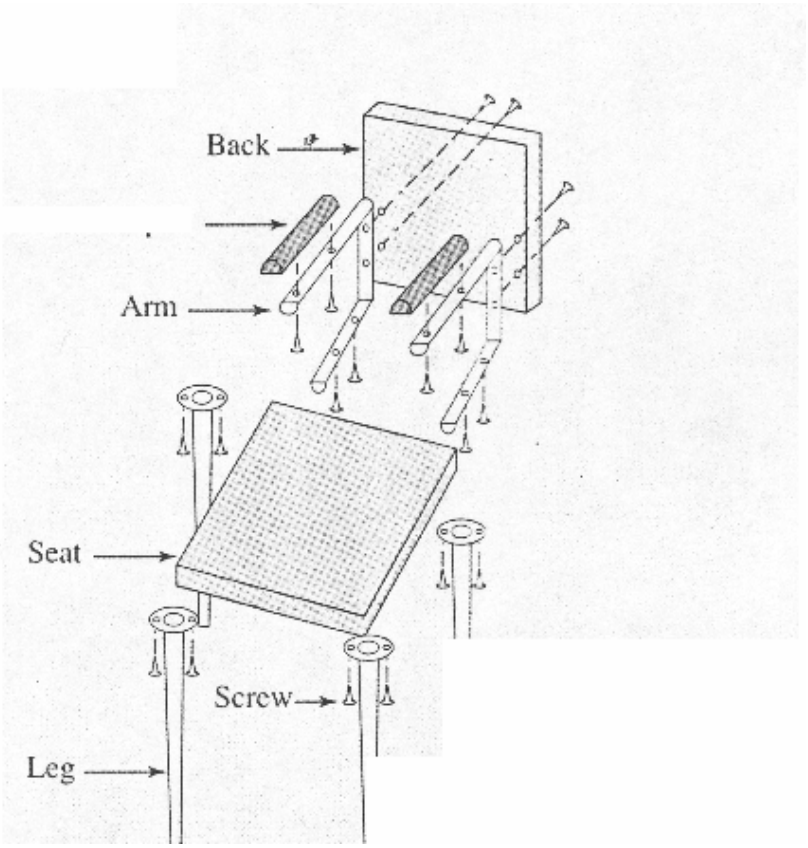
10. DOWN PAYMENT To buy a house, a couple was required to make a down payment of $11,400. What did the house sell for if this was 15% of the purchase price?

11. GREAT LAKES The Great Lakes have a total surface area of about 94,000 square miles. In square miles, Lake Huron covers 23,000, Lake Michigan 22,000, Lake Erie 10,000, and Lake Ontario 7,000. Find the surface area of Lake Superior.

Teamwork

1. PRODUCTION PLANNING

a. Decide on a product for which you will be the production planner. Make a detailed drawing of it like the one in Illustration 1.
b. For one component of your product, create a table that gives the number that should be ordered for production runs of 50, 100, 150, and 200 units. Do the same for a second component of your product using a bar graph, and for a third component using a line graph.
c. For each of the three components in part b, write an equation describing the number of components that need to be ordered for a production run of u units.



2. REAL NUMBERS Give some examples of situations Present your everyday life where you encounter the types of numbers listed below.

Whole Numbers
Zero
Negative Numbers
Fractions
Decimals

3. ORDER OF OPERATIONS To make a cake from a mix, the instructions must be followed carefully. Otherwise, the results can be disastrous. Think of two other multi-steps processes and explain why the steps must be performed in the proper order, or the outcome is adversely affected. Think of two processes in which the order in which the steps are per formed does not affect the outcome.

4. SUBTRACTION PROPERTY OF EQUALITY Check out a scale and some weights from your school's science department and use them as part of a class presentation to explain how the subtraction property of equality is used to solve the equation $x + 2 = 5$.

5. Suppose you were offered a job and could choose one of the following salaries:
a. $100 per hour or
b. 1 cent for the first day's work. 2 cents for the second day's work 4c for the third day's work and so on getting double the previous day's wages for every day in the month The next month you would start over at I c again. Which option would you choose for a month with 20 eight-hour working days?

6. A man uses two tickets a day for 20 days each month. If his tickets cost three for 25c, how much does he pay in a year?

Key for Essentials

3. a. The difference of 15 and 3 is 12.
4. a. $4 \cdot 9$; 4(9)
5. a. 8b

11. natural 8; whole 0, 8; integers 0, -12, 8; rational $\frac{-4}{5}$, 99.99, 0, -12, $4\frac{1}{2}$; irrational $\sqrt{2}$; real all

12. a. | -6| > |5|
13. a. 8^5
15. a. 6
16. a. h + 25
17.

x	$20x - x^3$
0	0
1	19
4	16

18. a. 36