Sierra College – Math 31 – **Exam #2**

NAME: Key score: /60

Be certain to show ALL of your work in a neat and organized manner giving explanations where needed. Anything that I cannot read or anything that I cannot follow because of lack of justification or lack of shown work, will get zero credit.

[5 points each]

Evaluate each of the following integrals or show that they are divergent.

1. $\int_0^2 \frac{x^2+x+1}{x^3+4x^2+5x+2}\,dx$ 7.4 #19

2. $\int_1^2 \frac{8\sqrt{x^2-1}}{x^4}\,dx$ 7.3 #5

3. $\int_0^1 (\arcsin x)^2\,dx$ 7.1 #22

4. $\int_0^6 \frac{x}{\sqrt{36-x^2}}\,dx$ 7.3 #6 (bounds – improper)

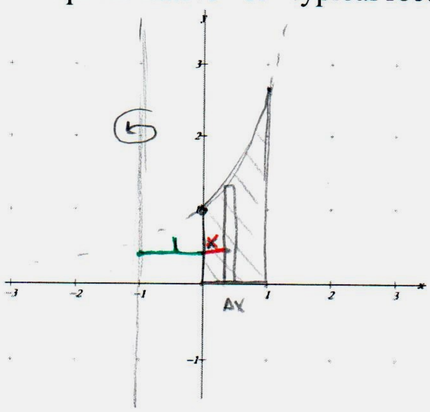
5. $\int_2^\infty \frac{4}{x^2+2x-3}\,dx$ 7.8 #18

6. $\int_0^a \frac{2a^2}{(a^2+x^2)^{3/2}}\,dx$ 7.3 #7

7. $\int_1^4 \frac{3}{\sqrt{\sqrt{x}-1}}\,dx$ 7.5 #64 (bounds – improper)

[10 points each]

8. Find the volume of the solid obtained by rotating the region (in the first quadrant) bounded by $y=e^x$ and $x=1$ about the line $x=-1$. Be sure you sketch the region on the grid provided below and clearly show a "representative" or "typical rectangle" (labeled appropriately).



7.1 # 62–64

Radius = $x+1$

9. Consider that $\int \sec x\,dx = \int \frac{1}{\cos x}\,dx = \int \frac{\cos x}{\cos^2 x}\,dx = \int \frac{\cos x}{1-\sin^2 x}\,dx$. Continue the integral from this point using the integration techniques we have studied to verify that $\int \sec x\,dx = \ln|\sec x + \tan x| + C$

10. Find p so that $\int_0^\infty \left(\frac{1}{\sqrt{x^2+1}} - \frac{p}{x+1}\right) dx$ is convergent. What does it equal?

7.8 # 79 & # 80

① Algebra 1st Note: $x=-1$ is a root of x^3+4x^2+5x+2

so...

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 5 & 2 \\ & \downarrow & -1 & -3 & -2 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$$x^3+4x^2+5x+2 = (x+1)(x^2+3x+2)$$
$$= (x+1)(x+1)(x+2)$$
$$= (x+2)(x+1)^2$$

$$\frac{x^2+x+1}{(x+2)(x+1)^2} = \frac{A}{x+2} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$x^2+x+1 = A(x+1)^2 + B(x+1)(x+2) + C(x+2)$$

if $x=-1 \rightarrow 1 = C(1) \rightarrow C=1$

if $x=-2 \rightarrow 3 = A(1) \rightarrow A=3$

if $x=0 \rightarrow 1 = 3+B(2)+2 \rightarrow -4=2B \rightarrow B=-2$

$$\int_0^2 \frac{x^2+x+1}{(x+2)(x+1)^2}\,dx = \int_0^2 \frac{3}{x+2} - \frac{2}{x+1} + \frac{1}{(x+1)^2}\,dx$$

$$= 3\ln|x+2| - 2\ln|x+1| - \frac{1}{x+1}\Big|_0^2 = \left(3\ln 4 - 2\ln 3 - \frac{1}{3}\right) - \left(3\ln 2 - 2\underbrace{\ln 1}_{0} - 1\right)$$

$$= \frac{2}{3} + \ln\left(\frac{4^3}{3^2\cdot 2^3}\right) = \boxed{\frac{2}{3} + \ln\left(\frac{8}{9}\right)}$$

② $\displaystyle\int_1^2 \frac{8\sqrt{x^2-1}}{x^4}\,dx$

$x=\sec\theta$ if $x=1 \rightarrow 1=\sec\theta \rightarrow \theta=0$

$dx = \sec\theta\tan\theta\,d\theta$ if $x=2 \rightarrow 2=\sec\theta \rightarrow \theta=\frac{\pi}{6}$

$$=8\int_0^{\pi/6} \frac{\sqrt{\sec^2\theta-1}}{\sec^4\theta}\sec\theta\tan\theta\,d\theta$$

$$=8\int_0^{\pi/6} \frac{\tan^2\theta}{\sec^3\theta}\,d\theta = 8\int_0^{\pi/6} \sin^2\theta\cos\theta\,d\theta$$

$u=\sin\theta$ if $\theta=0$, $u=0$

$du=\cos\theta$ if $\theta=\pi/6$, $u=\frac{\sqrt{3}}{2}$

$$=8\int_0^{\frac{\sqrt{3}}{2}} u^2\,du = \frac{8}{3}u^3\Big|_0^{\frac{\sqrt{3}}{2}} = \frac{8}{3}\left(\frac{\sqrt{3}}{2}\right)^3 = \boxed{\sqrt{3}}$$

③ $\int_0^1 (\arcsin x)^2\,dx$

Let's work with indefinite integral 1st...

$\int (\sin^{-1} x)^2\,dx$

$u = (\sin^{-1} x)^2 \qquad v = x$

$du = \dfrac{2(\sin^{-1} x)}{\sqrt{1-x^2}}\,dx \qquad dv = dx$

$= x(\sin^{-1} x)^2 - 2\int \dfrac{x \sin^{-1} x}{\sqrt{1-x^2}}\,dx$

$u = \sin^{-1} x \qquad v = -\sqrt{1-x^2}$

$du = \dfrac{1}{\sqrt{1-x^2}}\,dx \qquad dv = \dfrac{x}{\sqrt{1-x^2}}\,dx$

$= x(\sin^{-1} x)^2 - 2\left[-\sqrt{1-x^2}\,\sin^{-1} x + \int dx\right] = x(\sin^{-1} x)^2 + 2\sqrt{1-x^2}\,\sin^{-1} x - 2x + C$

Now back to original with bounds...

$\int_0^1 (\arcsin x)^2\,dx = x(\sin^{-1} x)^2 + 2\sqrt{1-x^2}\,\sin^{-1} x - 2x \Big|_0^1 = \left(\frac{\pi}{2}\right)^2 - 2 = \boxed{\frac{\pi^2}{4} - 2}$

④ $\int_0^6 \dfrac{x}{\sqrt{36-x^2}}\,dx$ Note @ $x=6$ the integrand has an asymptote!

$\lim_{b \to 6^-} \int_0^b \dfrac{x}{\sqrt{36-x^2}}\,dx$

$u = 36 - x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$\int \dfrac{x}{\sqrt{36-x^2}}\,dx = \int -\frac{1}{2}\,\frac{1}{\sqrt{u}}\,du$

$= -\cancel{\tfrac{1}{2}}\,\dfrac{u^{1/2}}{\cancel{1/2}} + C = -\sqrt{u} + C$

$= -\sqrt{36-x^2} + C$

$= \lim_{b \to 6^-} -\sqrt{36-x^2}\,\Big|_0^b$

$= \lim_{b \to 6^-} -\sqrt{36-b^2} + \sqrt{36-0} = -\underbrace{\sqrt{36-36}}_{\text{zero}} + \sqrt{36} = 0 + 6 = \boxed{6}$

(5) $\displaystyle\int_2^\infty \frac{4}{x^2+2x-3}\,dx$ Let's do Algebra 1st...

$$\frac{4}{(x+3)(x-1)} = \frac{A}{(x+3)} + \frac{B}{(x-1)} \rightarrow 4 = A(x-1)+B(x+3)$$

if $x=1 \rightarrow 4 = B(4) \rightarrow B=1$

if $x=-3 \rightarrow 4 = A(-4) \rightarrow A=-1$

$$= \lim_{b\to\infty} \int_2^b \frac{1}{x-1} - \frac{1}{x+3}\,dx = \lim_{b\to\infty} \ln|x-1| - \ln|x+3| \Big|_2^b$$

$$= \lim_{b\to\infty} \ln\left|\frac{b-1}{b+3}\right| - \ln\left(\frac{1}{5}\right) \overset{LH}{=} \lim_{b\to\infty} \overbrace{\ln\left|\frac{1}{1}\right|}^{\text{zero}} + \ln 5 = \boxed{\ln 5}$$

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(6) $\displaystyle\int_0^a \frac{2a^2}{(a^2+x^2)^{3/2}}\,dx$

$x = a\tan\theta$ if $x=0 \rightarrow 0 = a\tan\theta \rightarrow \theta = 0$

$dx = a\sec^2\theta\, d\theta$ if $x=a \rightarrow a = a\tan\theta \rightarrow \theta = \frac{\pi}{4}$

$$= \int_0^{\pi/4} \frac{2a^2\,(a\sec^2\theta)}{(a^2+a^2\tan^2\theta)^{3/2}}\,d\theta = 2\int_0^{\pi/4} \frac{\cancel{a^3}\sec^2\theta\,d\theta}{\cancel{a^3}\sec^3\theta\,d\theta} = \int_0^{\pi/4} 2\cos\theta\,d\theta$$

$$= 2\sin\theta\Big|_0^{\pi/4} = 2\left(\sin\left(\frac{\pi}{4}\right) - \sin(0)\right) = \cancel{2}\left(\frac{\sqrt{2}}{\cancel{2}}\right) = \boxed{\sqrt{2}}$$

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(7) $\displaystyle\int_1^4 \frac{3}{\sqrt{\sqrt{x}-1}}\,dx$ Note: Integrand has an asymptote @ $x=1$

Work with indefinite integral 1st...

$3\displaystyle\int \frac{1}{\sqrt{\sqrt{x}-1}}\,dx$

$u = \sqrt{x}-1 \longrightarrow u+1 = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx \rightarrow 2\sqrt{x}\,du = dx \longrightarrow 2(u+1)\,du = dx$

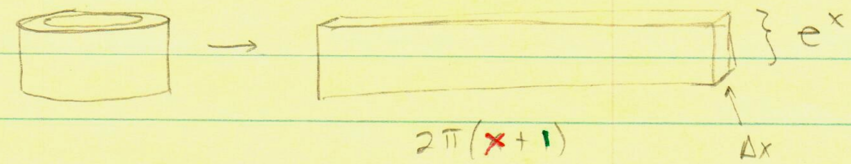
$$= 3\int \frac{1}{\sqrt{u}}\,2(u+1)\,du = 6\int u^{1/2} + u^{-1/2}\,du = 6\left(\frac{2}{3}u^{3/2} + 2u^{1/2}\right) + C$$

$$= 4\left(\sqrt{x}-1\right)^{3/2} + 12\left(\sqrt{x}-1\right)^{1/2} + C$$

$$= \lim_{a\to1^+} \int_a^4 \frac{3}{\sqrt{\sqrt{x}-1}}\,dx = \lim_{a\to1^+} 4(\sqrt{x}-1)^{3/2} + 12(\sqrt{x}-1)^{1/2}\Big|_a^4$$

$$= \lim_{a\to1^+} 16 - 4\underbrace{(\sqrt{a}-1)^{3/2}}_{\text{zero}} - 12\underbrace{(\sqrt{a}-1)^{1/2}}_{\text{zero}} = \boxed{16}$$

8



$$V = 2\pi \int_0^1 (x+1)\, e^x \, dx \qquad \begin{array}{ll} u = x+1 & v = e^x \\ du = dx & dv = e^x\, dx \end{array}$$

$$V = 2\pi \left((x+1)e^x \Big|_0^1 - \int_0^1 e^x \, dx \right) = 2\pi \left((x+1)\, e^x - e^x \right) \Big|_0^1$$

$$V = 2\pi \left(e^x x \Big|_0^1 \right) = 2\pi (e - 0) = \boxed{2\pi e \text{ (units)}^3}$$

9

$$\int \sec x \, dx = \int \frac{1}{\cos x} dx = \int \frac{\cos x}{\cos^2 x} dx = \int \frac{\cos x}{1-\sin^2 x} dx$$

$$\begin{array}{l} u = \sin x \\ du = \cos x \, dx \end{array} \quad = \int \frac{1}{1-u^2} du = \int \frac{1}{(1+u)(1-u)} du$$

$$\frac{1}{(1+u)(1-u)} = \frac{A}{1+u} + \frac{B}{1-u} \rightarrow \begin{array}{l} 1 = A(1-u) + B(1+u) \\ \text{if } u = 1 \rightarrow 1 = B(2) \rightarrow B = 1/2 \\ \text{if } u = -1 \rightarrow 1 = A(2) \rightarrow A = 1/2 \end{array}$$

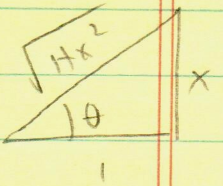
$$= \int \frac{1/2}{1+u} + \frac{1/2}{1-u} du = \frac{1}{2}\left(\ln|1+u| - \ln|1-u| \right) + C$$

$$= \frac{1}{2} \ln \left| \frac{1+u}{1-u} \right| + C = \frac{1}{2} \ln \left| \frac{(1+u)(1+u)}{1-u^2} \right| + C = \ln \left| \sqrt{\frac{(1+u)^2}{1-u^2}} \right| + C$$

$$= \ln \left| \frac{1+u}{\sqrt{1-u^2}} \right| + C = \ln \left| \frac{1+\sin x}{\sqrt{1-\sin^2 x}} \right| + C$$

$$= \ln \left| \frac{1+\sin x}{\cos x} \right| + C = \boxed{\ln | \sec x + \tan x | + C}$$

(10) Lets work with $\int \frac{1}{\sqrt{x^2+1}} - \frac{p}{x+1}\,dx$ 1st...



$x = \tan\theta$
$dx = \sec^2\theta\,d\theta$

$$\int \frac{1}{\sqrt{\tan^2\theta+1}} \sec^2\theta\,d\theta - p\int \frac{1}{x+1}\,dx$$

$$= \int \sec\theta\,d\theta - p\ln|x+1| + C = \ln|\sec\theta+\tan\theta| - \ln|(x+1)^p| + C$$

$$= \ln\left|\sqrt{1+x^2}+x\right| - \ln|(x+1)^p| + C = \ln\left|\frac{\sqrt{1+x^2}+x}{(x+1)^p}\right| + C$$

Now back to original...

$$\int_0^\infty \frac{1}{\sqrt{x^2+1}} - \frac{p}{x+1}\,dx = \lim_{b\to\infty} \ln\left|\frac{\sqrt{1+x^2}+x}{(x+1)^p}\right| \Bigg|_0^b$$

$$= \lim_{b\to\infty} \ln\left|\frac{\sqrt{1+b^2}+b}{(b+1)^p}\right| - \overbrace{\ln|1|}^{\text{zero}} = \lim_{b\to\infty} \ln\left(\frac{\sqrt{1+b^2}+b}{(b+1)^p}\right)$$

Note: the "growth" of the numerator is linear and so to force a valid limit we must have $p=1$ to match in the denominator

$$= \lim_{b\to\infty} \ln\left(\frac{\sqrt{\frac{1}{b^2}+1}+1}{\left(1+\frac{1}{b}\right)}\right) = \ln\left(\frac{\sqrt{1}+1}{1}\right) = \boxed{\ln 2}$$

Sierra College – Math 31 – **Exam #2 (Take Home Question)**

NAME: ____

(DUE Monday 10/2) You may use your notes, textbook and other research material. You may "DISCUSS" these problems with each other, but you may NOT work with tutors, instructors or anyone who is not in this course. Make certain that your final write-up is your own work and be certain to show and explain ALL of your work. I expect very clean final solutions since this is a take-home.

Consider the integral

$$\int_1^2 x^x dx$$

- Determine (using the error bound formulas) the number of regions required to estimate this integral accurate to within .001 using one of the techniques covered in class (Midpoint, Trapezoid or Simpsons).
- Approximate the given integral with the appropriate number of regions and hence the correct accuracy.

To begin we have $\int_1^2 x^x dx = \int_a^b f(x)\, dx$ and so $f(x) = x^x$ and $a = 1$ and $b = 2$

For our error bound formulas we will need to calculate derivatives of $f(x)$ so we begin with those first…

$$f(x) = x^x$$
$$\ln\left(f(x)\right) = \ln\left(x^x\right)$$
$$\ln\left(f(x)\right) = x\ln x$$
$$\frac{1}{f(x)} f'(x) = (x)\left(\frac{1}{x}\right) + (1)(\ln x)$$
$$\frac{1}{f(x)} f'(x) = 1 + \ln x$$
$$f'(x) = f(x)(1 + \ln x)$$
$$\boxed{f'(x) = x^x(1 + \ln x)}$$

Now for the second derivative we have…

$$
\begin{aligned}
f''(x) &= \left(x^x\right)\frac{d}{dx}(1+\ln x)+\left[\frac{d}{dx}\left(x^x\right)\right](1+\ln x) \\
&= \left(x^x\right)\left(\frac{1}{x}\right)+\left[x^x(1+\ln x)\right](1+\ln x) \\
&= \boxed{x^x\left(\frac{1}{x}+(1+\ln x)^2\right)}
\end{aligned}
$$

Note also that $f''(x)=x^x\left(\frac{1}{x}+(1+\ln x)^2\right)=x^x\frac{1}{x}+x^x(1+\ln x)^2=x^{x-1}+x^x(1+\ln x)^2$

Since x^x is increasing on the interval $[1,2]$ AND x^{x-1} is increasing on the interval $[1,2]$ AND $(1+\ln x)^2$ is increasing on the interval $[1,2]$ AND we know that $f''(x)$ is just the sum of a bunch of increasing functions we conclude that the maximum value the second derivative can have on this interval will occur when $x=2$.

$$
f''(2)=(2)^{(2)}\left(\frac{1}{(2)}+(1+\ln(2))^2\right)=4\left(\frac{1}{2}+1+2\ln 2+(\ln 2)^2\right)=6+8\ln 2+4(\ln 2)^2=6+8\ln 2+2^2(\ln 2)^2
$$

$$
=6+8\ln 2+2^2(\ln 2)^2=6+8\ln 2+(2\ln 2)^2=6+8\ln 2+(\ln 4)^2=K\approx 13.4669895
$$

So looking at the error bound for the Trapezoid Rule we have $|E_T|<\frac{K(b-a)^3}{12n^2}$ and we must have…

$$
\begin{aligned}
&\frac{K(b-a)^3}{12n^2}<.001 \\
&\frac{K(2-1)^3}{12n^2}<\frac{1}{1000} \\
&\frac{K}{12n^2}<\frac{1}{1000} \\
&\frac{12n^2}{K}>1000 \\
&12n^2>1000K \\
&n^2>\frac{1000K}{12} \\
&n>\sqrt{\frac{1000K}{12}}\approx\sqrt{\frac{1000(13.4669895)}{12}}\approx 33.49998694 \\
&n\ge 34
\end{aligned}
$$

So if we decide to use the Trapezoid Rule for our estimation we will need at least 34 regions.

Now let's look at the error bound for the Midpoint Rule we have $\left|E_M\right| < \dfrac{K(b-a)^3}{24n^2}$ and we must have…

$$\frac{K(b-a)^3}{24n^2} < .001$$
$$\frac{K\left(2-1\right)^3}{24n^2} < \frac{1}{1000}$$
$$\frac{K}{24n^2} < \frac{1}{1000}$$
$$\frac{24n^2}{K} > 1000$$
$$24n^2 > 1000K$$
$$n^2 > \frac{1000K}{24}$$
$$n > \sqrt{\frac{1000K}{24}} \approx \sqrt{\frac{1000(13.4669895)}{24}} \approx 23.68806794$$
$$n \geq 24$$

So if we decide to use the Midpoint Rule for our estimation we will need at least 24 regions

Based on this I think most of us would like MUCH less regions so we will proceed with Simpson's method which does require the 4$^{\text{th}}$ derivative.

Recall that $f''(x) = x^x\left(\dfrac{1}{x}+\left(1+\ln x\right)^2\right)$ so…

$$f^3(x) = \left(x^x\right)\frac{d}{dx}\left(\frac{1}{x}+\left(1+\ln x\right)^2\right)+\left(\frac{1}{x}+\left(1+\ln x\right)^2\right)\frac{d}{dx}\left(x^x\right)$$
$$= \left(x^x\right)\left(-\frac{1}{x^2}+2\left(1+\ln x\right)\frac{1}{x}\right)+\left(\frac{1}{x}+\left(1+\ln x\right)^2\right)\left(x^x\left(1+\ln x\right)\right)$$
$$= x^x\left(\left(-\frac{1}{x^2}+2\left(1+\ln x\right)\frac{1}{x}\right)+\left(\frac{1}{x}+\left(1+\ln x\right)^2\right)\left(1+\ln x\right)\right)$$
$$= x^x\left(\left(-\frac{1}{x^2}+\frac{2}{x}+\frac{2}{x}\ln x\right)+\frac{1}{x}\left(1+\ln x\right)+\left(1+\ln x\right)^3\right)$$
$$= x^x\left(-\frac{1}{x^2}+\frac{2}{x}+\frac{2}{x}\ln x+\frac{1}{x}+\frac{1}{x}\ln x+\left(1+\ln x\right)^3\right)$$
$$= \boxed{x^x\left(-\frac{1}{x^2}+\frac{3}{x}+\frac{3}{x}\ln x+\left(1+\ln x\right)^3\right)}$$

$$f^4(x)=\left(x^x\right)\frac{d}{dx}\left(-\frac{1}{x^2}+\frac{3}{x}+\frac{3}{x}\ln x+\left(1+\ln x\right)^3\right)+\left(-\frac{1}{x^2}+\frac{3}{x}+\frac{3}{x}\ln x+\left(1+\ln x\right)^3\right)\frac{d}{dx}\left(x^x\right)$$

$$=x^x\left(\frac{2}{x^3}-\frac{3}{x^2}+\left(\frac{3}{x}\right)\left(\frac{1}{x}\right)+\left(-\frac{3}{x^2}\right)\left(\ln x\right)+3\left(1+\ln x\right)^2\left(\frac{1}{x}\right)\right)+\left(-\frac{1}{x^2}+\frac{3}{x}+\frac{3}{x}\ln x+\left(1+\ln x\right)^3\right)\left(x^x\left(1+\ln x\right)\right)$$

$$=x^x\left(\frac{2}{x^3}-\frac{3}{x^2}+\frac{3}{x^2}+\left(-\frac{3}{x^2}\right)\left(\ln x\right)+3\left(1+\ln x\right)^2\left(\frac{1}{x}\right)+\left(-\frac{1}{x^2}+\frac{3}{x}+\frac{3}{x}\ln x+\left(1+\ln x\right)^3\right)\left(1+\ln x\right)\right)$$

$$=x^x\left(\frac{2}{x^3}-\frac{3}{x^2}\left(\ln x\right)+\frac{3}{x}\left(1+\ln x\right)^2+-\frac{1}{x^2}\left(1+\ln x\right)+\frac{3}{x}\left(1+\ln x\right)+\frac{3}{x}\ln x\left(1+\ln x\right)+\left(1+\ln x\right)^4\right)$$

$$=x^x\left(\frac{2}{x^3}-\frac{3}{x^2}\left(\ln x\right)+\frac{3}{x}\left(1+\ln x\right)^2+-\frac{1}{x^2}\left(1+\ln x\right)+\frac{3}{x}\left(1+\ln x\right)+\frac{3}{x}\ln x+\frac{3}{x}\left(\ln x\right)^2+\left(1+\ln x\right)^4\right)$$

$$=\boxed{x^{x-3}\left(2-3x\left(\ln x\right)+3x^2\left(1+\ln x\right)^2-x\left(1+\ln x\right)+3x^2\left(1+\ln x\right)+3x^2\ln x+3x^2\left(\ln x\right)^2+x^3\left(1+\ln x\right)^4\right)}$$

As before since all of the "parts" of this 4th derivative are increasing we know that $f^4(x)$ is increasing so we conclude that the maximum value the 4th derivative can have on this interval will occur when $x=2$.

$$f^4(2)=(2)^{2-3}\left(2-3(2)\left(\ln(2)\right)+3(2)^2\left(1+\ln(2)\right)^2-(2)\left(1+\ln(2)\right)+3(2)^2\left(1+\ln(2)\right)+3(2)^2\ln(2)+3(2)^2\left(\ln(2)\right)^2+(2)^3\left(1+\ln(2)\right)^4\right)$$

$$=\frac{1}{2}\left(2-6\ln(2)+12\left(1+\ln(2)\right)^2-2-2\ln(2)+12+12\ln(2)+12\ln(2)+12\left(\ln(2)\right)^2+8\left(1+\ln(2)\right)^4\right)$$

$$=1-3\ln 2+6\left(1+2\ln 2+(\ln 2)^2\right)-1-\ln 2+6+12\ln 2+6(\ln 2)^2+4\left(1+4\ln 2+6(\ln 2)^2+4(\ln 2)^3+(\ln 2)^4\right)$$

$$=1-3\ln 2+6+12\ln 2+6(\ln 2)^2-1-\ln 2+6+12\ln 2+6(\ln 2)^2+4+16\ln 2+24(\ln 2)^2+16(\ln 2)^3+4(\ln 2)^4$$

$$=4(\ln 2)^4+16(\ln 2)^3+36(\ln 2)^2+36\ln 2+16=K\approx 64.50134183$$

So looking at the error bound for the Simpson's Rule we have $\left|E_S\right|<\dfrac{K(b-a)^5}{180n^4}$ and we must have…

$$\frac{K(b-a)^5}{180n^4}<.001$$

$$\frac{K(2-1)^5}{180n^4}<\frac{1}{1000}$$

$$\frac{K}{180n^4}<\frac{1}{1000}$$

$$\frac{180n^4}{K}>1000$$

$$n^4>\frac{1000K}{180}$$

$$n>\sqrt[4]{\frac{1000K}{180}}\approx\sqrt[4]{\frac{1000(64.50134183)}{180}}\approx 4.350849502$$

Since the number of regions for Simpson's rule must be even we round this UP to $n=6$ regions. Now onto some calculations…

According to Simpson's rule $\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0)+4f(x_1)+2f(x_2)+4f(x_3)+2f(x_4)+4f(x_5)+f(x_6)\right]$

where $\Delta x = \frac{b-a}{n} = \frac{2-1}{6} = \frac{1}{6}$ So… $\frac{\Delta x}{3} = \frac{\frac{1}{6}}{3} = \frac{1}{18}$ and so we get…

$$\int_1^2 x^x\,dx \approx \frac{1}{18}\left[f(1)+4f\left(1+\frac{1}{6}\right)+2f\left(1+\frac{2}{6}\right)+4f\left(1+\frac{3}{6}\right)+2f\left(1+\frac{4}{6}\right)+4f\left(1+\frac{5}{6}\right)+f\left(1+\frac{6}{6}\right)\right]$$

$$=\frac{1}{18}\left[f(1)+4f\left(\frac{7}{6}\right)+2f\left(\frac{4}{3}\right)+4f\left(\frac{3}{2}\right)+2f\left(\frac{5}{3}\right)+4f\left(\frac{11}{6}\right)+f(2)\right]$$

$$=\frac{1}{18}\left[1^1+4\left(\frac{7}{6}\right)^{\frac{7}{6}}+2\left(\frac{4}{3}\right)^{\frac{4}{3}}+4\left(\frac{3}{2}\right)^{\frac{3}{2}}+2\left(\frac{5}{3}\right)^{\frac{5}{3}}+4\left(\frac{11}{6}\right)^{\frac{11}{6}}+2^2\right]$$

$$\approx \frac{1}{18}\left[1+4(1.197028768)+2(1.467523222)+4(1.837117307)+2(2.342868515)+4(3.038150594)+4\right]$$

$$= 2.050553897 \approx \boxed{2.051 \pm .001}$$